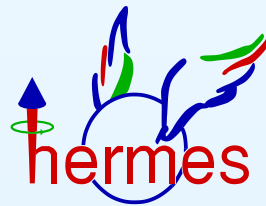


***Signals for transverse-momentum-dependent
distribution and fragmentation functions
observed at the HERMES experiment***

Markus Diefenthaler

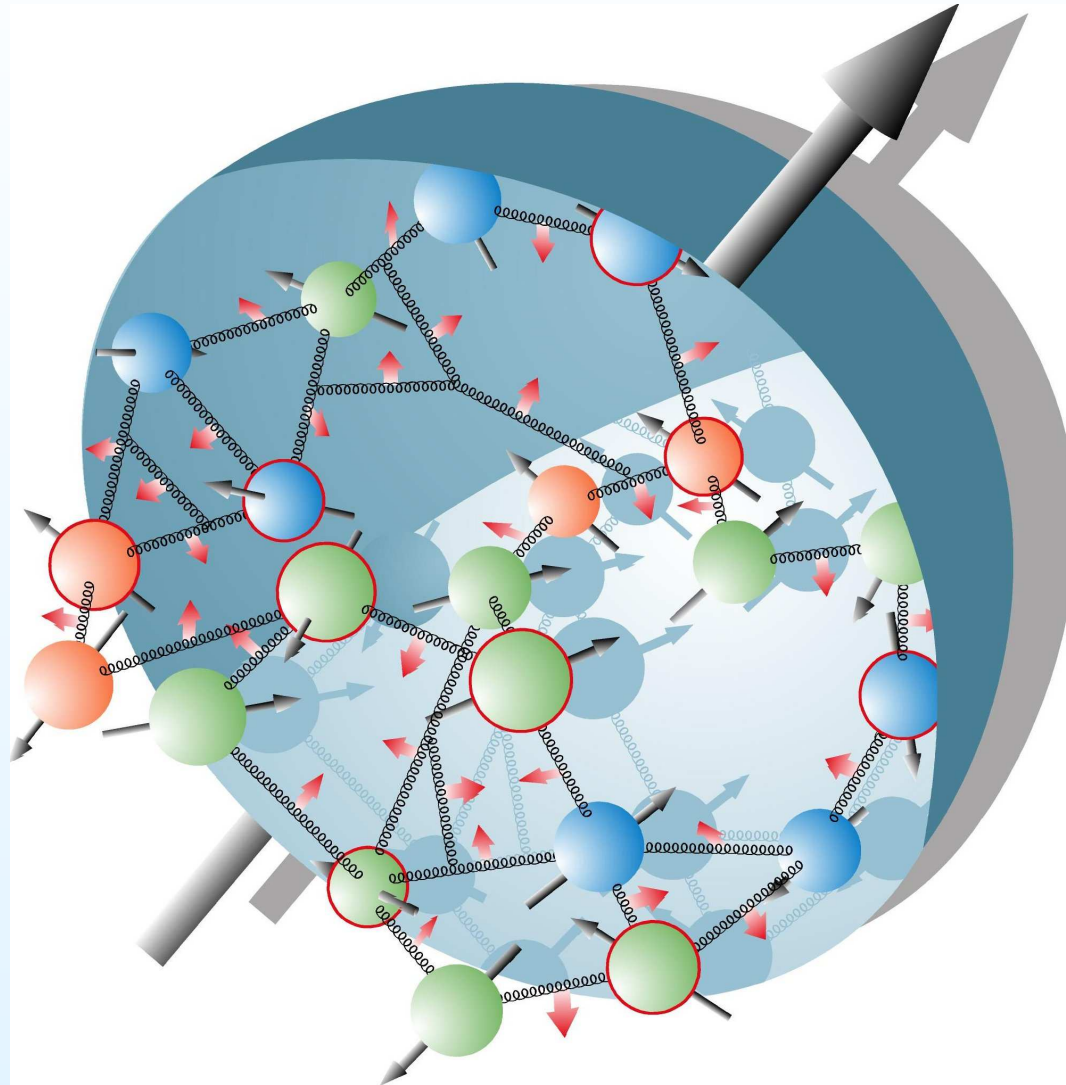


ILLINOIS
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN



on behalf of the hermes collaboration

The spin structure of the nucleon:

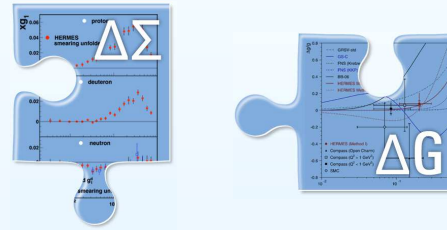


The HERMES legacy:

Longitudinal spin phenomena (1995–2000):

- angular momentum sum rule:

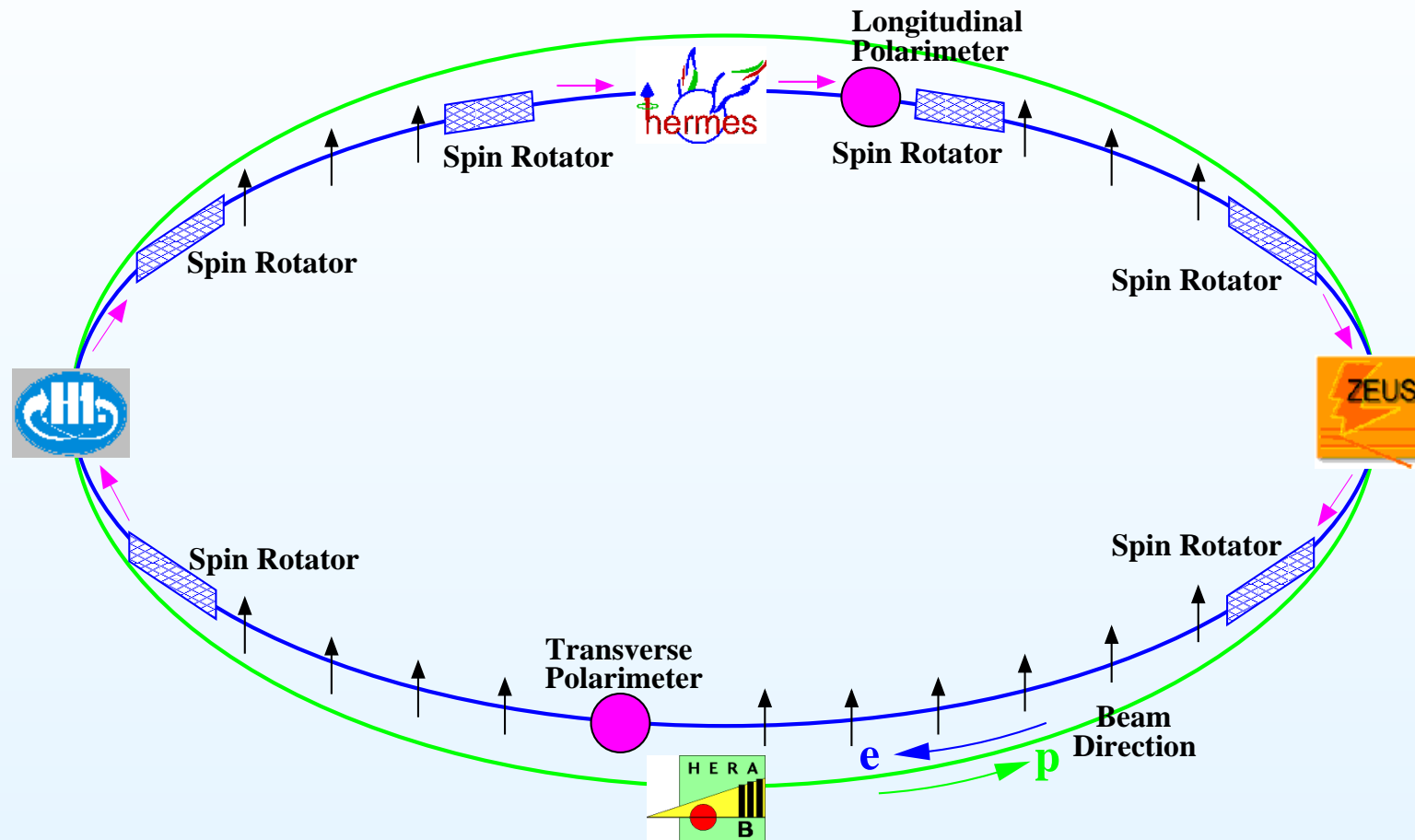
$$\frac{s_z^N}{\hbar} = \frac{1}{2} = \frac{1}{2} \Delta\Sigma + L_q + \Delta G + L_g$$



Transverse spin phenomena (2002–2005):

- investigation of σ_{UU} , σ_{UL} , σ_{UT} , σ_{LU}
- transversity measurements
- spin-orbit correlations via TMD measurements
 - ↳ Sivers function $f_{1T}^{\perp,q}$
 - ↳ Boer-Mulders function $h_1^{\perp,q}$
 - ↳ pretzelosity $h_{1T}^{\perp,q}$

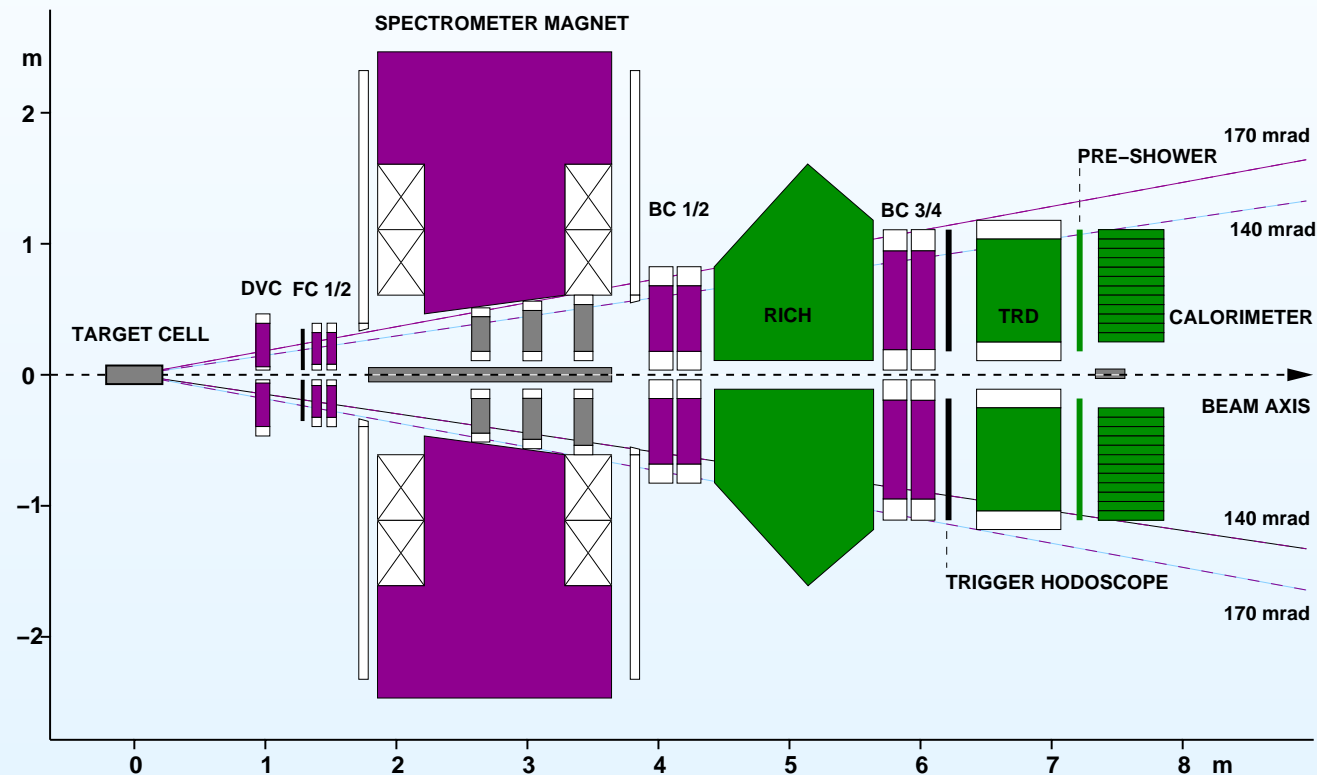
The HERMES polarised scattering experiment:



- longitudinally polarised e^+ and e^- beam of HERA
- $\sqrt{s} \approx 7 \text{ GeV}$

The HERMES polarised scattering experiment:

- (un)polarised **gas target** internal to the HERA storage ring
- background-free measurements from highly polarised nucleons



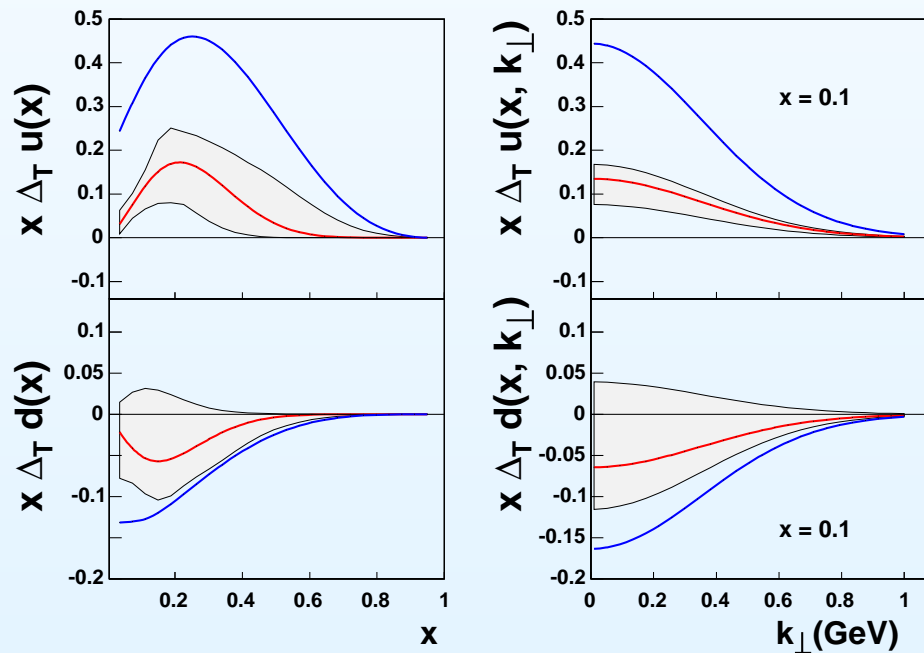
- very clean lepton-hadron separation and hadron identification
- well-suited for **measurements of azimuthal asymmetries**

The hunt for the chiral-odd transversity distribution:

- complete description of quark momentum and spin:

$$\Phi(x) = \frac{1}{2} \left\{ f_1^q(x) \not{P} + \lambda_N g_1^q(x) \gamma_5 \not{P} + h_1^q(x) \not{P} \gamma_5 \not{S}_\perp \right\}$$

- extraction by Anselmino et al., **Phys.Rev.D75:054032,2007**:

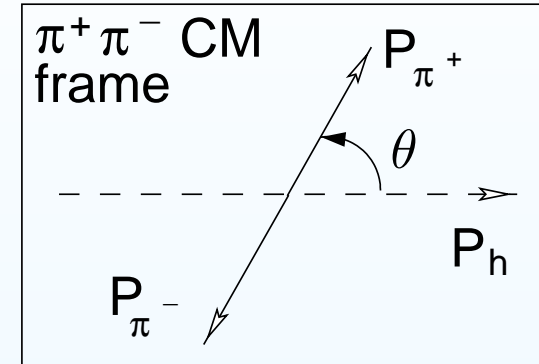
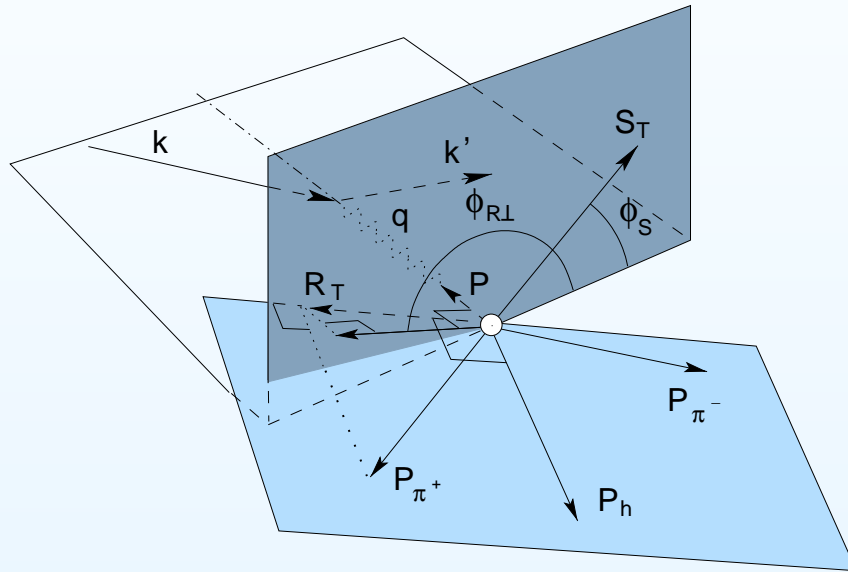


using data from:

- HERMES
- COMPASS
- BELLE

The semi-inclusive production of $\pi^+\pi^-$ pairs:

transverse SSA: $S_q \cdot (p_q \times R)$



$$P_h \equiv P_{\pi^+} + P_{\pi^-}$$

$$R \equiv \frac{P_{\pi^+} - P_{\pi^-}}{2}$$

$$R_T \equiv R - (R \cdot \hat{P}_h) \hat{P}_h$$

azimuthal angles ϕ_S and ϕ_{R_\perp} :

$$\phi_S \equiv \frac{(\mathbf{q} \times \mathbf{k}) \cdot \mathbf{S}_T}{|(\mathbf{q} \times \mathbf{k}) \cdot \mathbf{S}_T|} \arccos \left(\frac{(\mathbf{q} \times \mathbf{k}) \cdot (\mathbf{q} \times \mathbf{S}_T)}{|\mathbf{q} \times \mathbf{k}| |\mathbf{q} \times \mathbf{S}_T|} \right)$$

$$\phi_{R_\perp} \equiv \frac{(\mathbf{q} \times \mathbf{k}) \cdot \mathbf{R}_T}{|(\mathbf{q} \times \mathbf{k}) \cdot \mathbf{R}_T|} \arccos \left(\frac{(\mathbf{q} \times \mathbf{k}) \cdot (\mathbf{q} \times \mathbf{R}_T)}{|\mathbf{q} \times \mathbf{k}| |\mathbf{q} \times \mathbf{R}_T|} \right)$$

SSA in semi-inclusive $\pi^+\pi^-$ production:

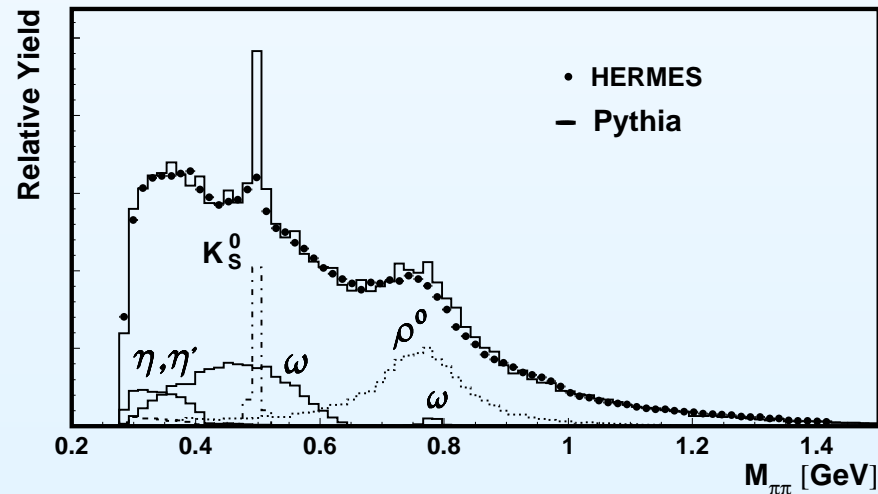
- **Fourier and Legendre expansion:**

$$A_{UT}^{\sin(\phi_{R\perp} + \phi_S) \sin \theta} \sim \frac{\sum_q e_q^2 h_1^q(x) H_{1,q}^{\triangleleft,sp}(z, M_{\pi\pi})}{\sum_q e_q^2 f_1^q(x) D_{1,q}(z, M_{\pi\pi})}$$

- focus on **sp- and pp-interference** ($M_{\pi\pi} < 1.5$ GeV):

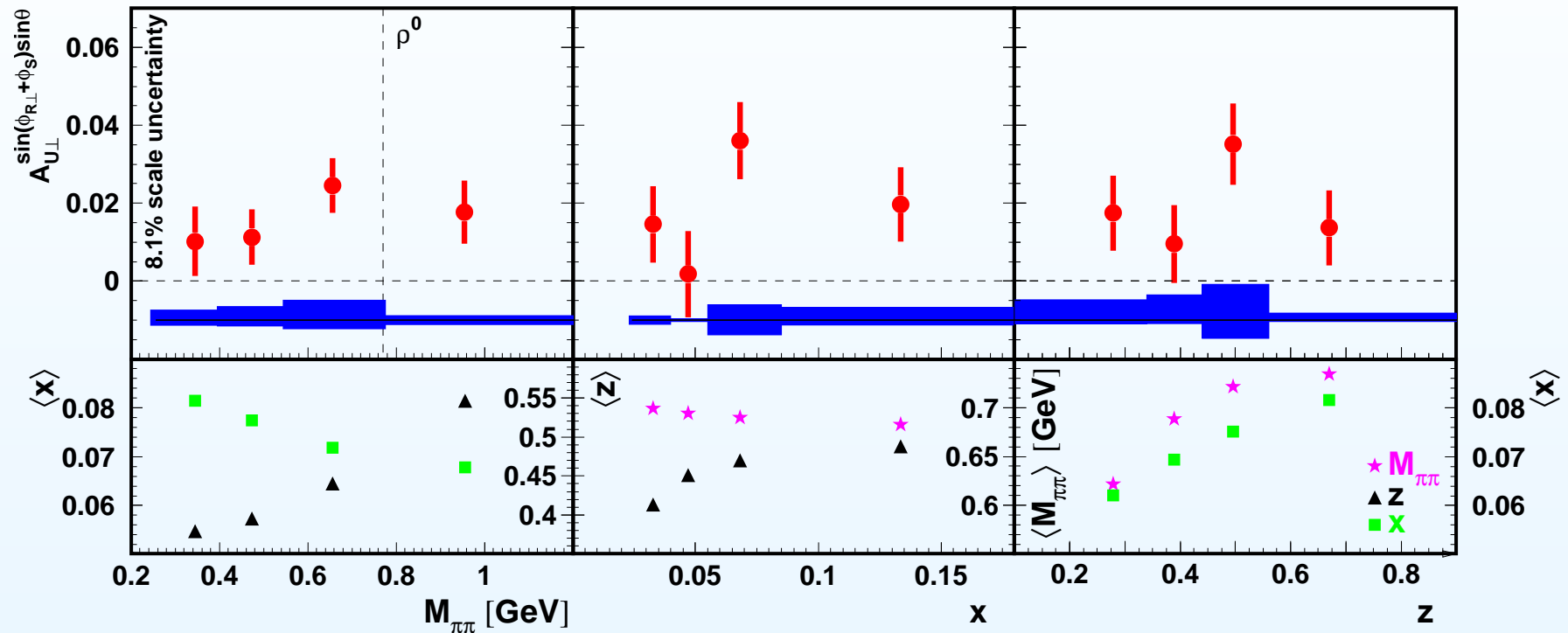
$$\rightarrow D_{1,q} \simeq D_{1,q} + D_{1,q}^{sp} \cos \theta + D_{1,q}^{pp} \frac{1}{4} (3 \cos^2 \theta - 1)$$

$$\rightarrow H_{1,q}^{\triangleleft} \simeq H_{1,q}^{\triangleleft,sp} + H_{1,q}^{\triangleleft,pp} \cos \theta$$



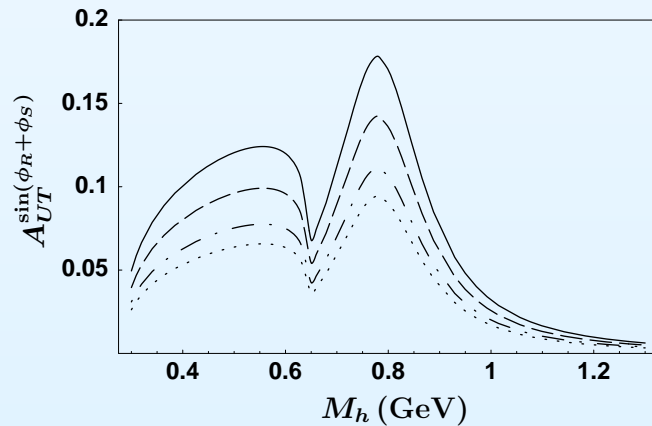
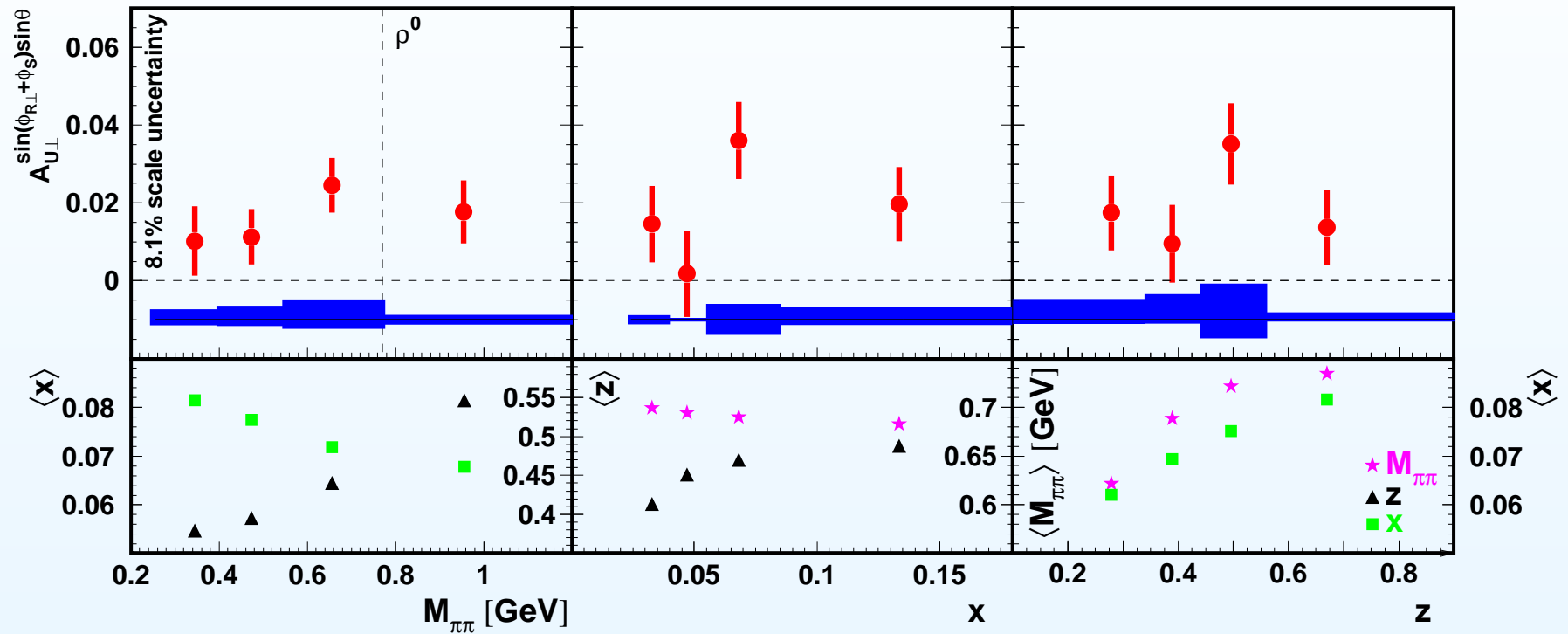
- symmetrisation around $\theta = \pi/2 \rightarrow D_{1,q}^{sp}$ and $H_{1,q}^{\triangleleft,pp}$ drop out

Results on SSA in semi-inclusive $\pi^+\pi^-$ production:



- $A_{U\perp}^{\sin(\phi_{R\perp} + \phi_S) \sin\theta} = 0.018 \pm 0.005_{\text{stat}} \pm 0.002_{\text{b-scan}} + 0.004_{\text{acc}}$
- additional 8.1% scale uncertainty (target polarisation)
- first evidence for $H_{1,q}^{\triangleleft}$
- transversity can be studied in dihadron production

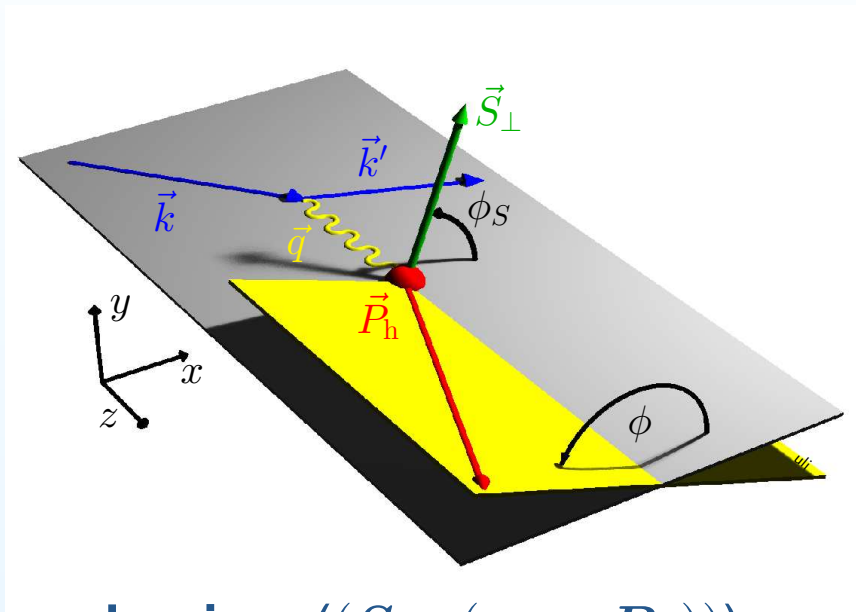
Results on SSA in semi-inclusive $\pi^+\pi^-$ production:



Bacchetta, Radici
(Phys.Rev.D74:114007,2006)

Transversity measurement in single-hadron production:

- observation of **azimuthal asymmetry** $A_{UT}(\phi, \phi_S)$:

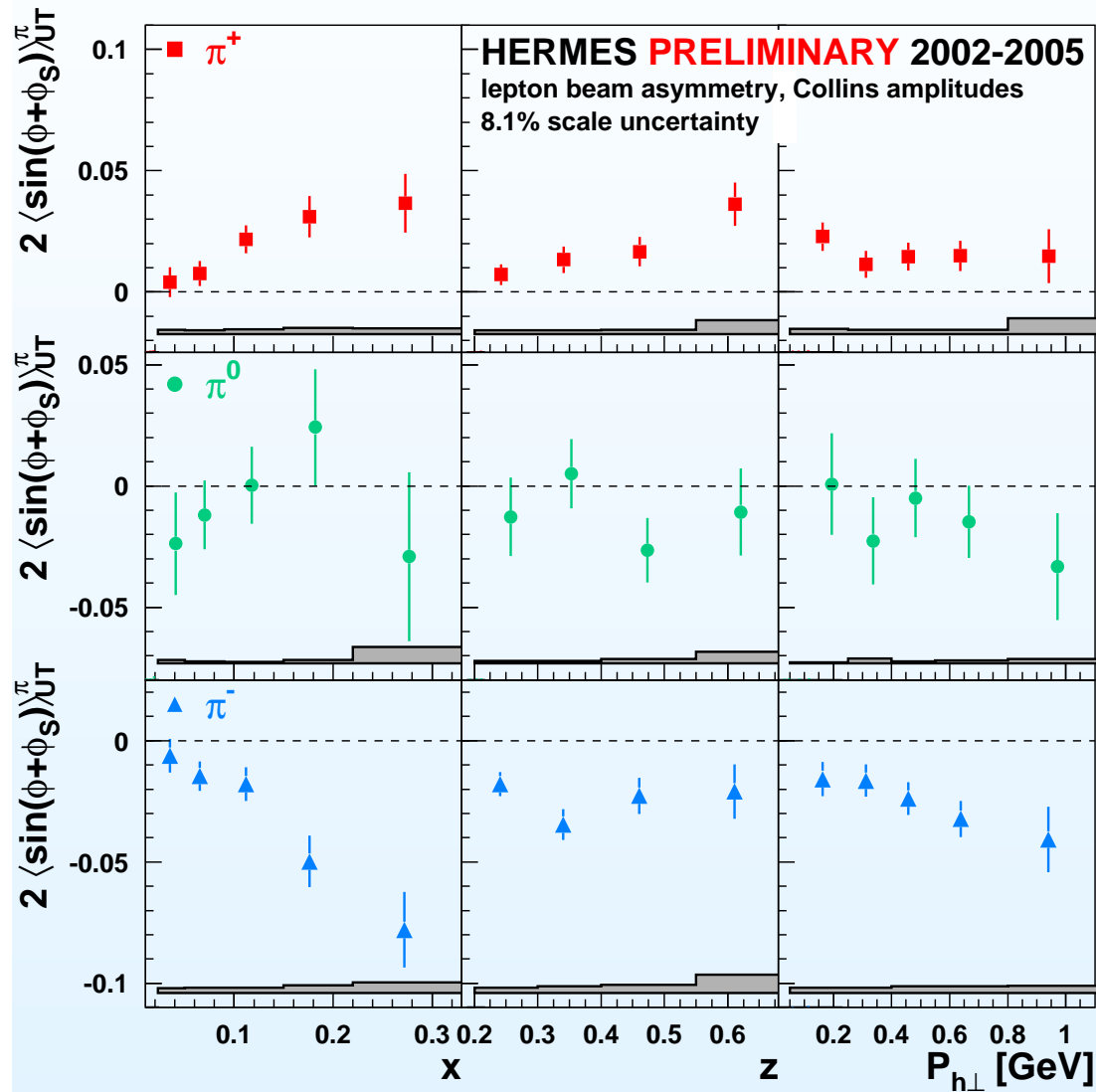


- due to **Collins mechanism** ($(S_q \cdot (p_q \times P_h))$)
- **Fourier decomposition of $\sigma_{U\perp}$** including:

$$2\langle \sin(\phi + \phi_S) \rangle_{UT} = \frac{\sum_q e_q^2 h_1^q(x, p_T^2) \otimes_{\mathcal{W}} H_1^{\perp, q}(z, K_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, K_T^2)},$$

$$\sin(\phi - \phi_S), \sin(3\phi - \phi_S), \sin(\phi_S), \sin(2\phi - \phi_S), \sin(2\phi + \phi_S).$$

The Collins amplitudes for pions:



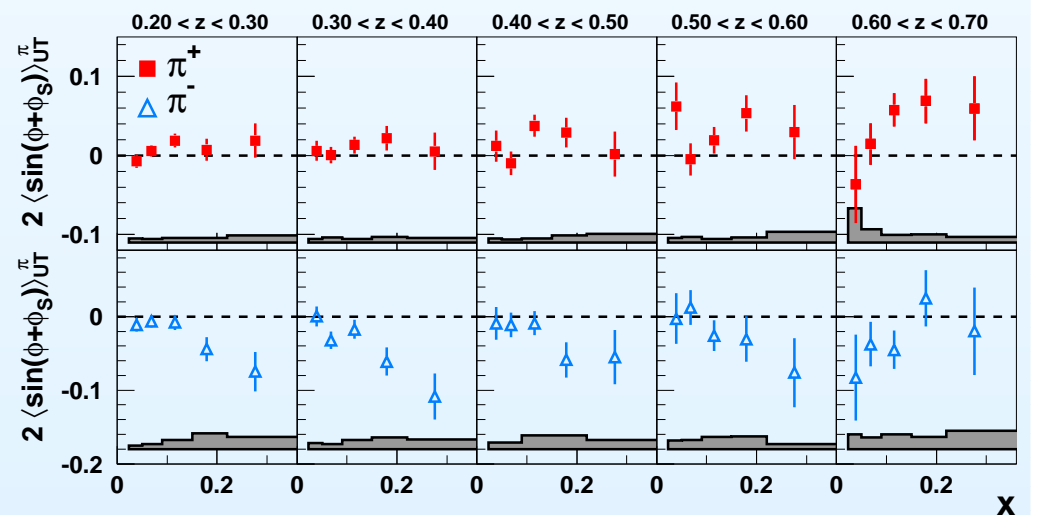
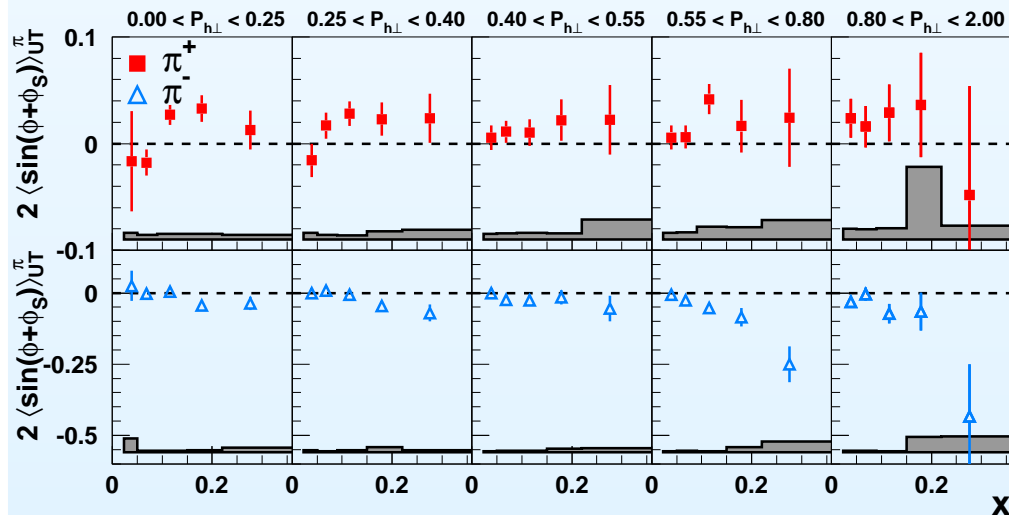
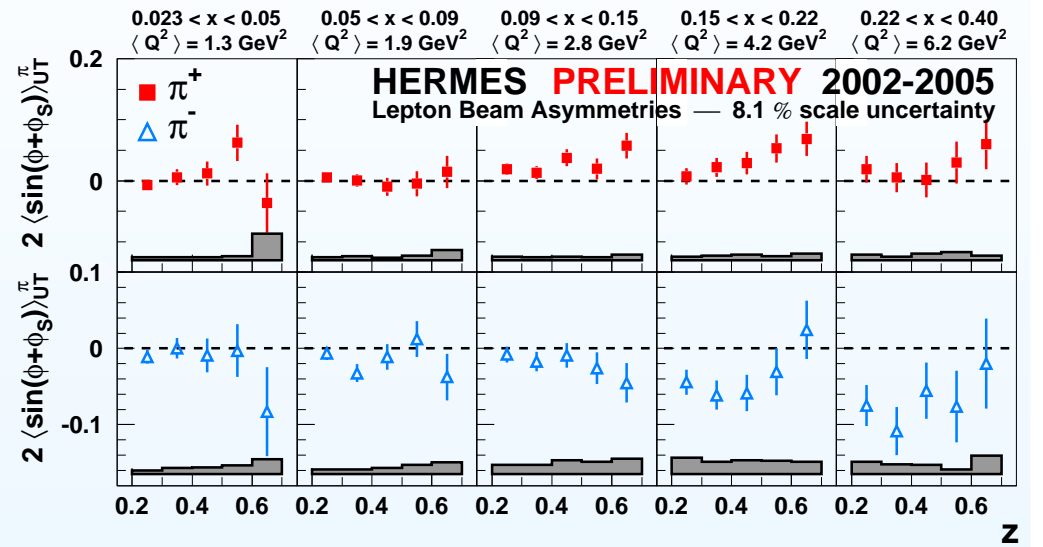
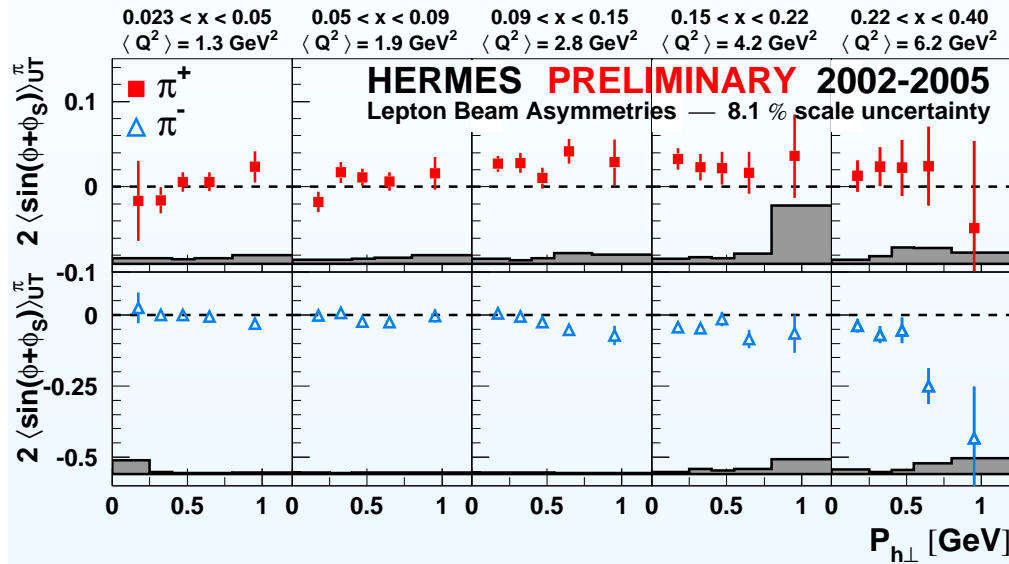
Results of the Collins amplitude:

$$h_1^q(x) \otimes H_1^{\perp q}(z)$$

from 2002–2005 data:

- positive amplitudes for π^+
- large negative π^- amplitudes unexpected
- $H_1^{\perp, \text{unfav}}(z) \approx -H_1^{\perp, \text{fav}}(z)$
- isospin symmetry of π -mesons fulfilled

The kinematic dependence of the Collins amplitudes:

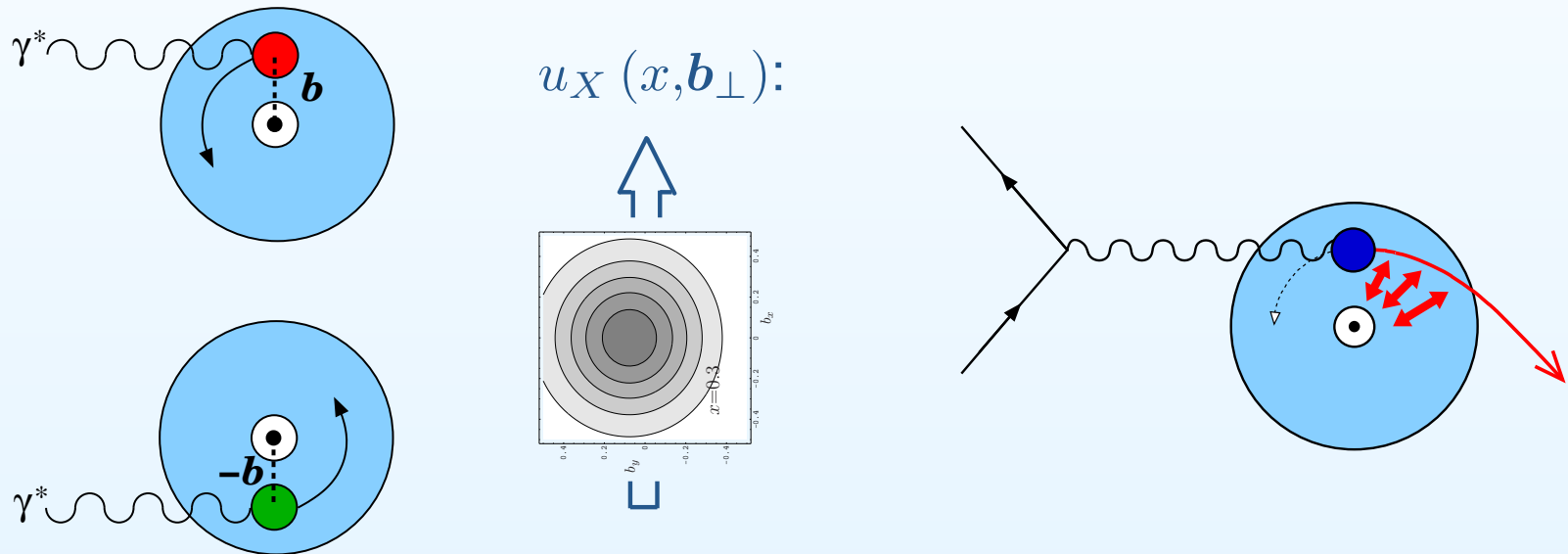


Evidence for naive-T-odd distribution functions:

- **naive time reversal odd (naive-T-odd) functions**
- involve interference of amplitudes with different helicities
 - ↳ suppressed in perturbative QCD
 - ↳ assigned to distribution and fragmentation functions
- **associated with spin/orbit effects ($S \cdot (P_1 \times P_2)$)**
- observation of the naive-T-odd **Sivers function** f_{1T}^\perp
- observation of the naive-T-odd **Boer-Mulders function** h_1^\perp

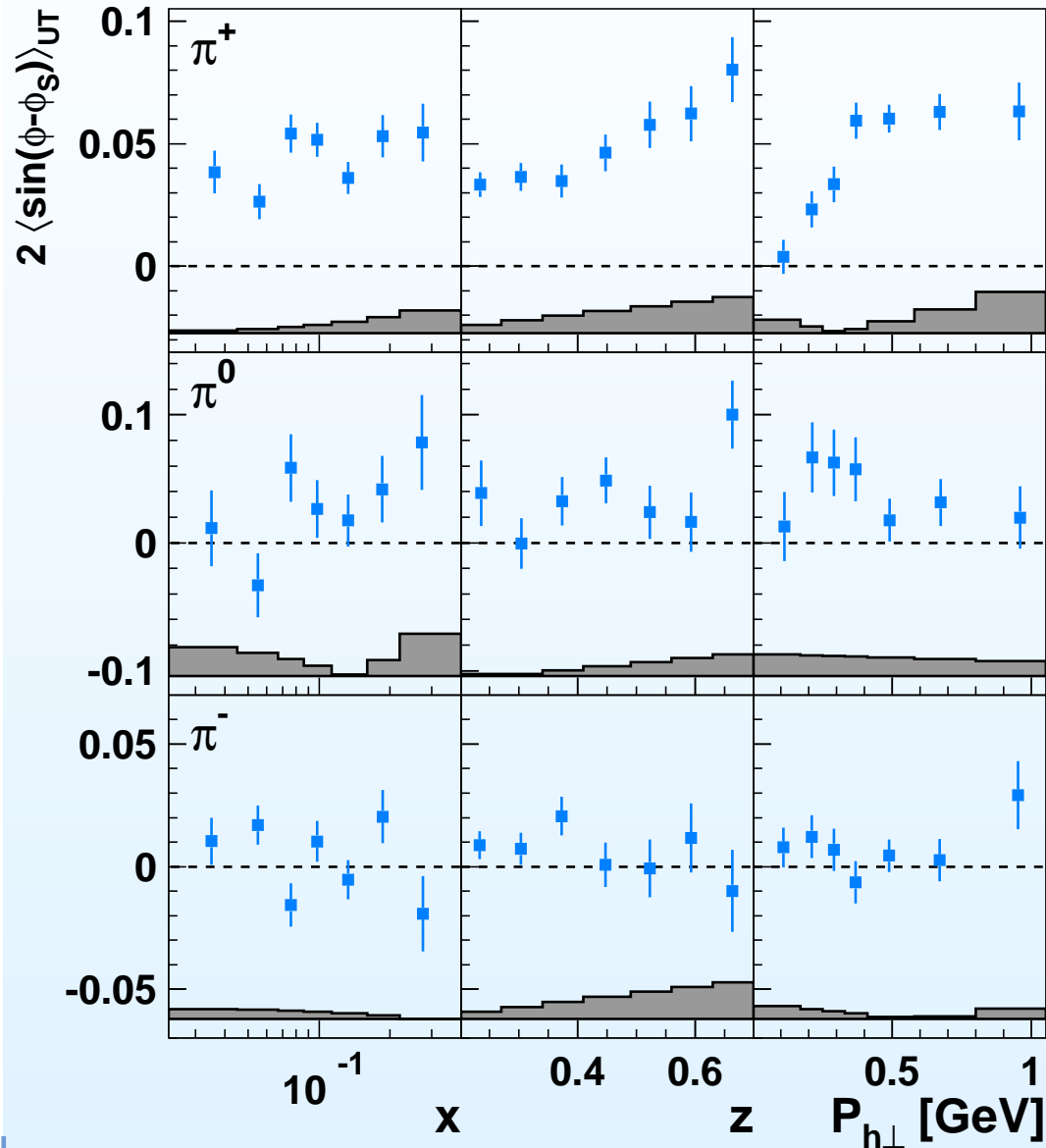
The Sivers mechanism:

- non-zero **Sivers distribution** f_{1T}^\perp involves non-zero Compton amplitude $N^{\uparrow} q^\uparrow \rightarrow N^{\downarrow} q^\uparrow$
- **orbital angular momentum of quarks:** (M. Burkardt, (Phys.Rev.D66:114005,2002))



- **SSA due to Sivers mechanism** ($S_q \cdot (P \times p_q)$)

The Sivers amplitudes for π -mesons:



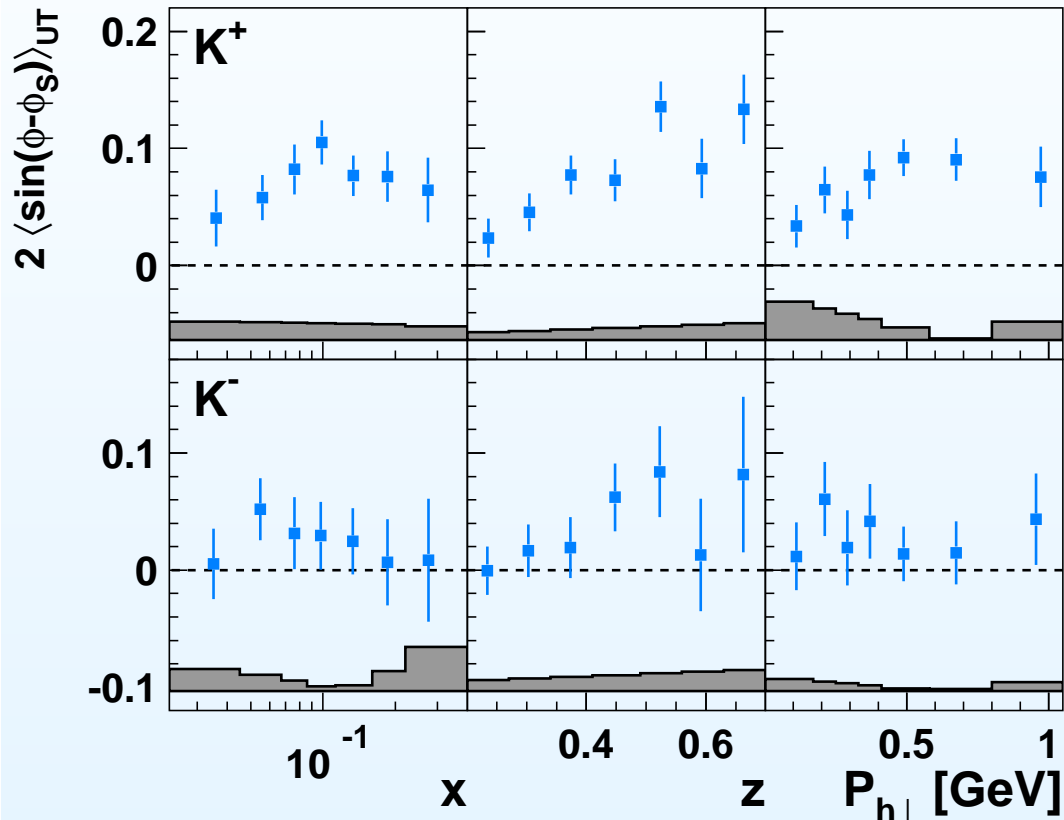
Results for Sivers amplitude:

$$f_{1T}^{\perp q}(x) \otimes D_1^q(z).$$

from 2002–2005 data:

- significantly positive for π^+
 $\hookrightarrow f_{1T}^{\perp,u} < 0, L_z^u > 0$
- significantly positive for π^0
- consistent with zero for π^-
 $\hookrightarrow f_{1T}^{\perp,d} > 0?$
- increase with z for π^+ and π^0
- $P_{h\perp} > 0.4$ GeV: saturation for π^+
- $P_{h\perp} \rightarrow 0.0$ GeV: linear decrease
- isospin symmetry fulfilled

The Sivers amplitudes for charged K -mesons:



Results for Sivers amplitude:

$$f_{1T}^{\perp q}(x) \otimes D_1^q(z).$$

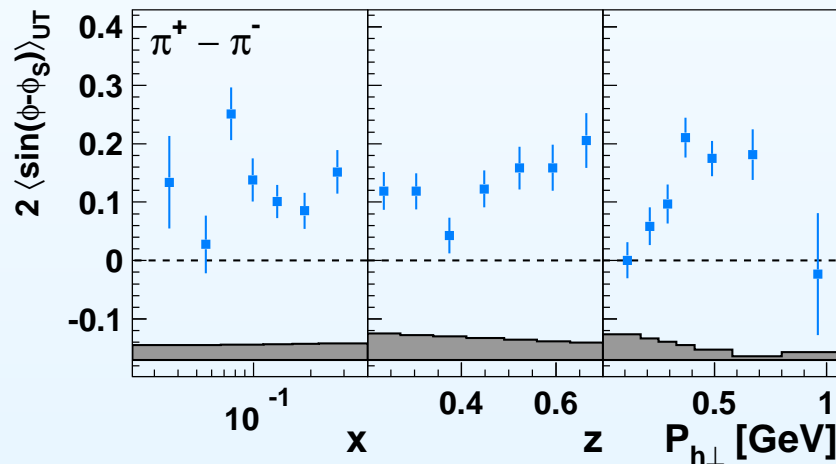
from 2002–2005 data:

- significantly positive for K^+
 $\rightarrow f_{1T}^{\perp,u} < 0, L_z^u > 0$
- significantly positive for K^-
- increase with z
- $P_{h\perp} > 0.4$ GeV: saturation for K^+
- $P_{h\perp} \rightarrow 0.0$ GeV: linear decrease

Pion-difference Sivers amplitudes:

- suppress ρ^0 contribution by extraction of pion-difference SSA:

$$A_{UT}^{\pi^+ - \pi^-}(\phi, \phi_S) \equiv \frac{1}{|\mathbf{S}_T|} \frac{(\sigma_{U\uparrow}^{\pi^+} - \sigma_{U\uparrow}^{\pi^-}) - (\sigma_{U\downarrow}^{\pi^+} - \sigma_{U\downarrow}^{\pi^-})}{(\sigma_{U\uparrow}^{\pi^+} - \sigma_{U\uparrow}^{\pi^-}) + (\sigma_{U\downarrow}^{\pi^+} - \sigma_{U\downarrow}^{\pi^-})}$$



- significantly positive
 $\rightarrow f_{1T}^{\perp,u} < 0, L_z^u > 0$
- increase with z
- saturation for $P_{h\perp} > 0.4 \text{ GeV}$
- linear decrease for $P_{h\perp} \rightarrow 0.0 \text{ GeV}$

- possible interpretation in terms of valence-quark distributions:

$$A_{UT}^{\pi^+ - \pi^-} = \frac{f_{1T}^{\perp,d_v} - 4f_{1T}^{\perp,u_v}}{f_1^{d_v} - 4f_1^{u_v}}$$

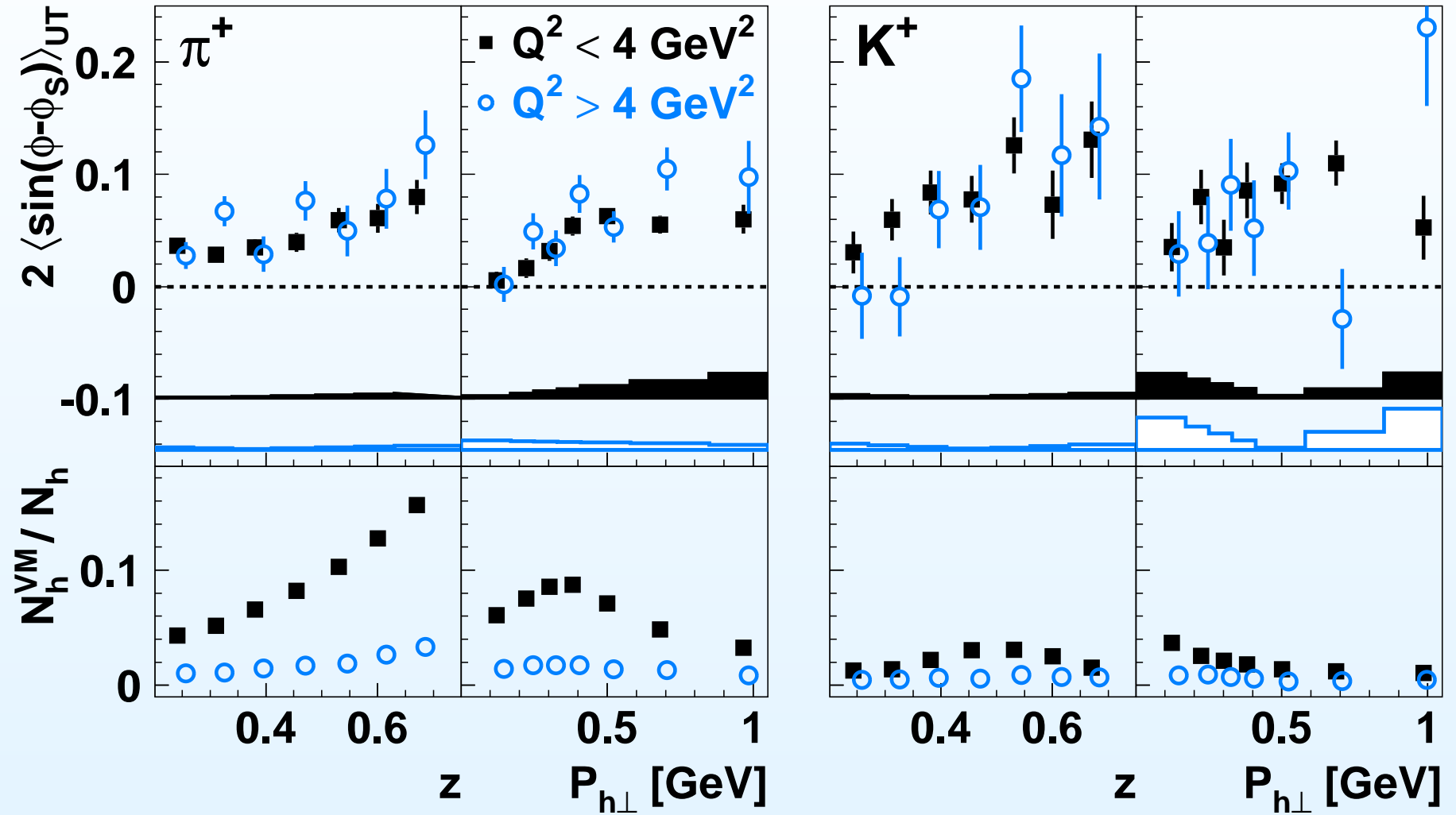
The role of higher twist terms:

- **Sivers amplitude:**

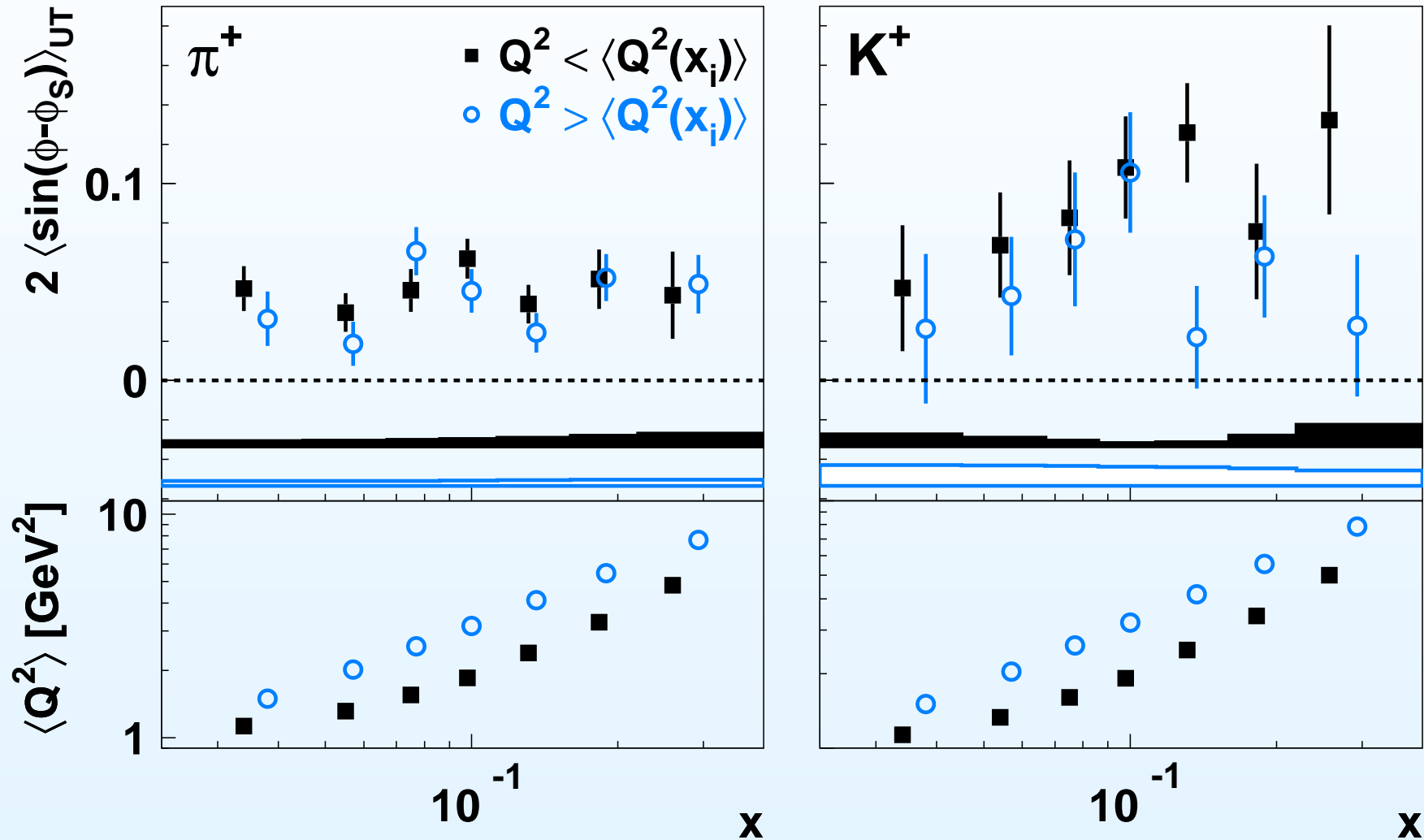
$$2 \langle \sin(\phi - \phi_S) \rangle_{\text{UT}} \propto F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)}$$

- $F_{UT,T}^{\sin(\phi - \phi_S)} = \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} f_{1T}^\perp D_1 \right]$
- $F_{UT,L}^{\sin(\phi - \phi_S)} = 0$ (leading twist and subleading twist accuracy)
 - $\frac{q_T^2}{Q^2}$ -suppressed compared to $F_{UT,T}$
 - can be generated by α_s -corrections at high transverse momentum

Examination of vector-meson contribution:



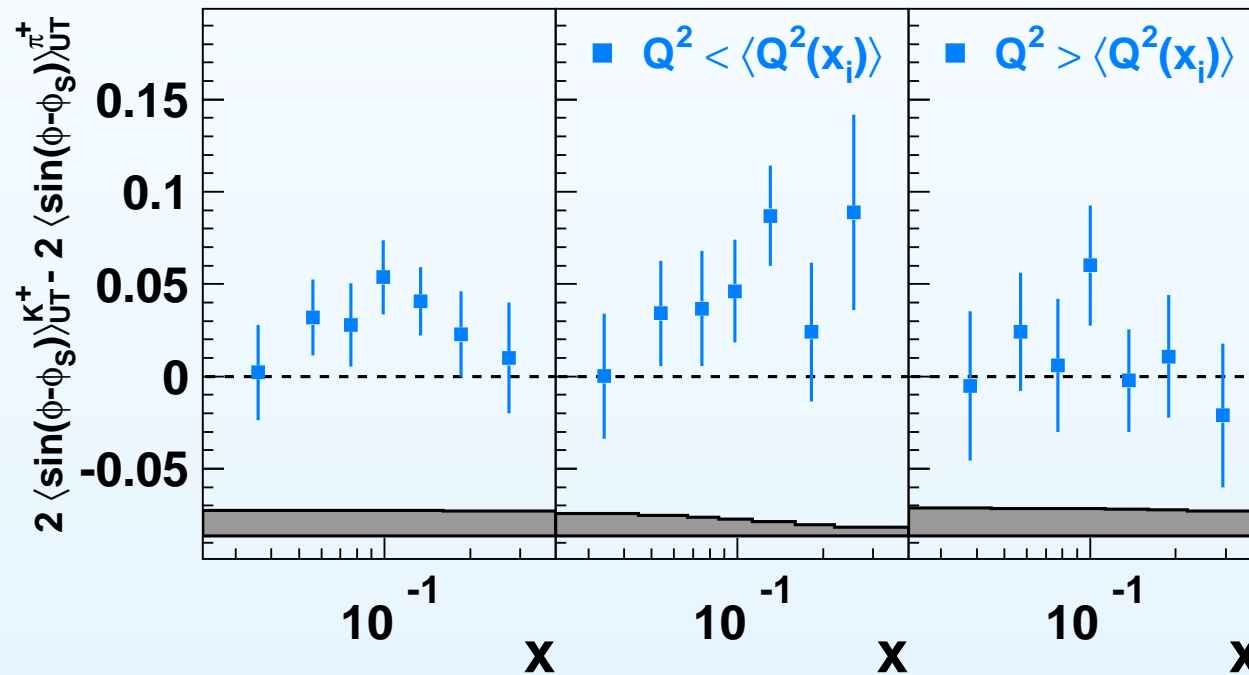
Examination of other $1/Q^2$ -suppressed contributions:



hint of Q^2 -dependence for K^+ amplitudes

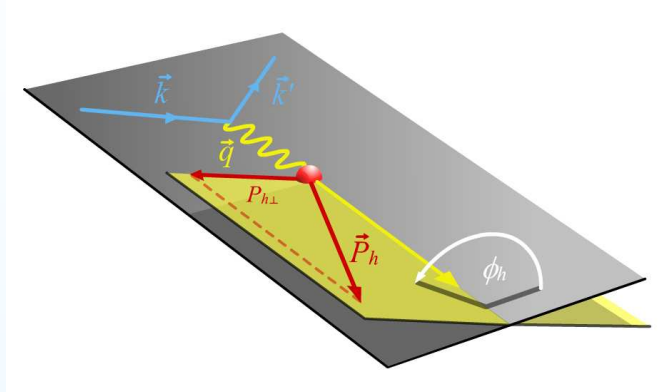
Sivers amplitudes for K^+ and π^+ :

- **u -quark dominance:** $2\langle \sin(\phi - \phi_S) \rangle_{\text{UT}}^{\pi^+} \sim 2\langle \sin(\phi - \phi_S) \rangle_{\text{UT}}^{K^+}$
- **difference in K^+ and π^+ Sivers amplitudes:**



- significant role of other quark flavours?
- higher twist effects in kaon-production?

Signals for unmeasured Boer-Mulders function h_1^\perp :



Azimuthal modulations of σ_{UU} :

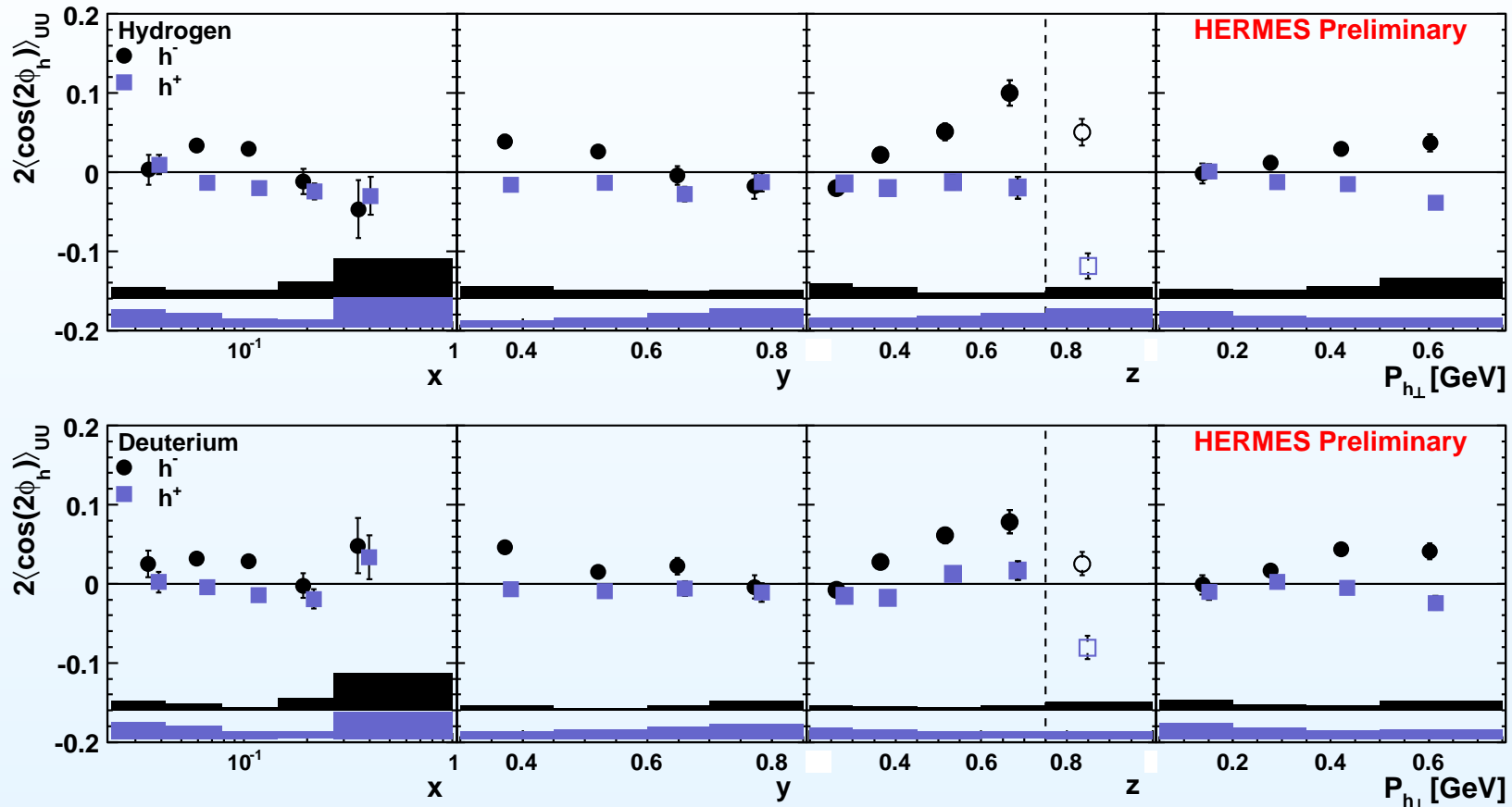
- leading-twist 2 $\langle \cos(2\phi) \rangle_{UU}$
 - sensitive to **Boer-Mulders function** ($h_1^\perp \otimes H_1^\perp$)
- subleading-twist 2 $\langle \cos(\phi) \rangle_{UU}$
 - sensitive to **Cahn effect** ($f_1 \otimes D_1$) and $h_1^\perp \otimes H_1^\perp$

Fully differential analysis $(x, y, z, P_{h\perp}, \phi)$

➔ correction for finite acceptance, QED radiation, detector smearing

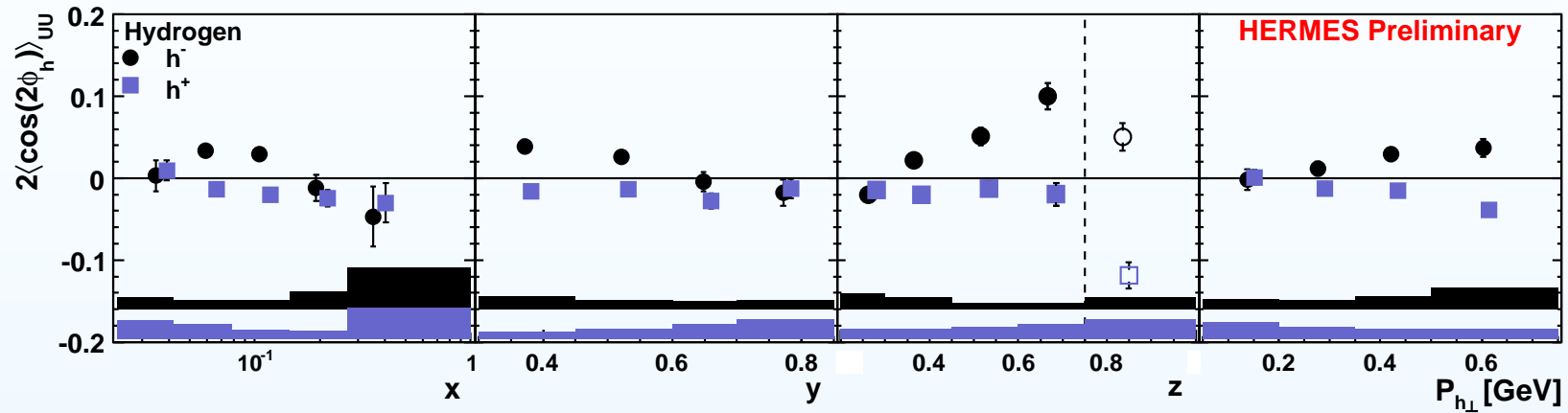
hydrogen (2000, 2006) and deuterium (2000, 2005) data

Results for $2 \langle \cos(2\phi) \rangle_{UU}$:

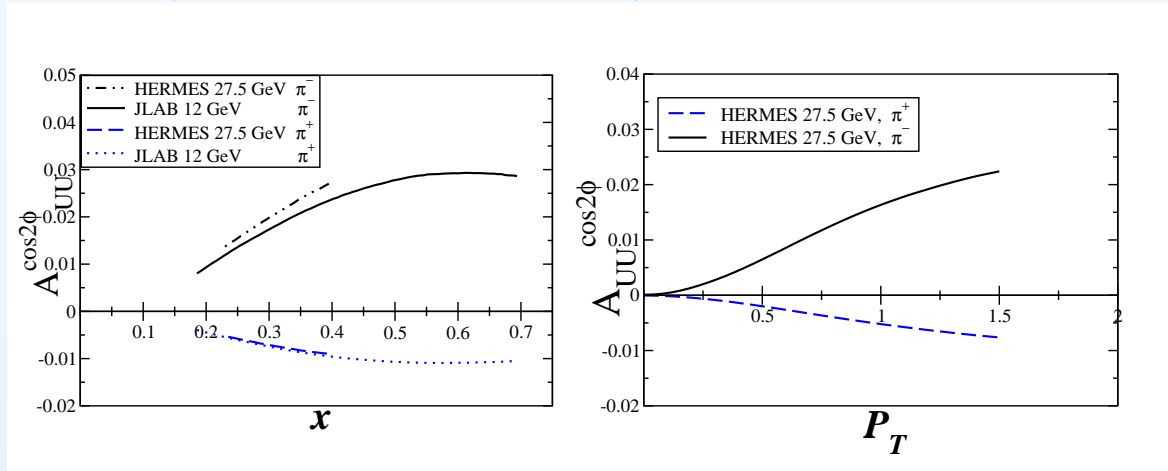


- significantly positive for h^-
- slightly negative for h^+
- $h_1^{\perp,u} = h_1^{\perp,d}$ or $h_1^{\perp,u} = -h_1^{\perp,d}$

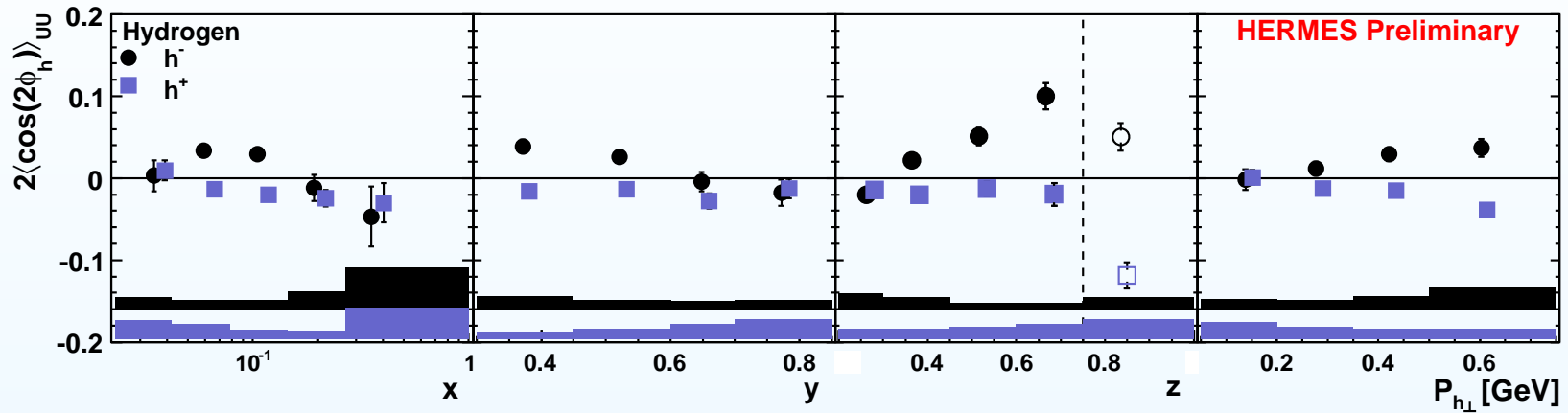
Clear signal for Boer-Mulders function?:



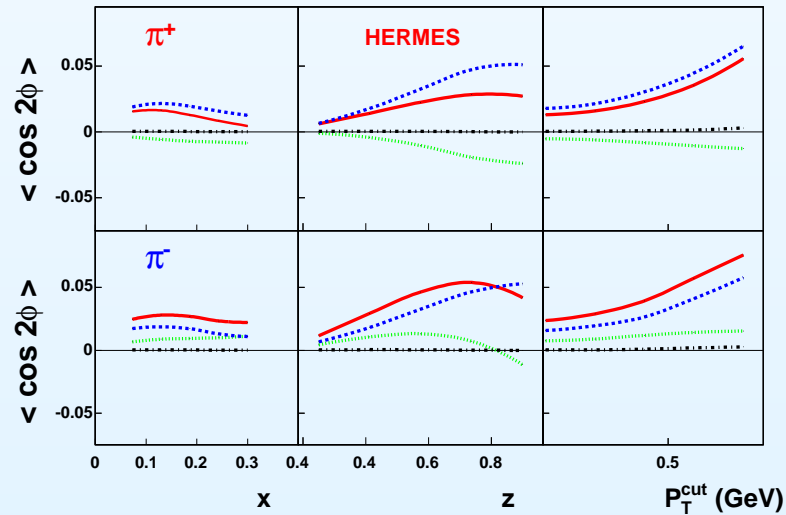
model by Gamberg, Goldstein, Schlegel, [Phys.Rev.D77:094016,2007](#)



Clear signal for Boer-Mulders function?:

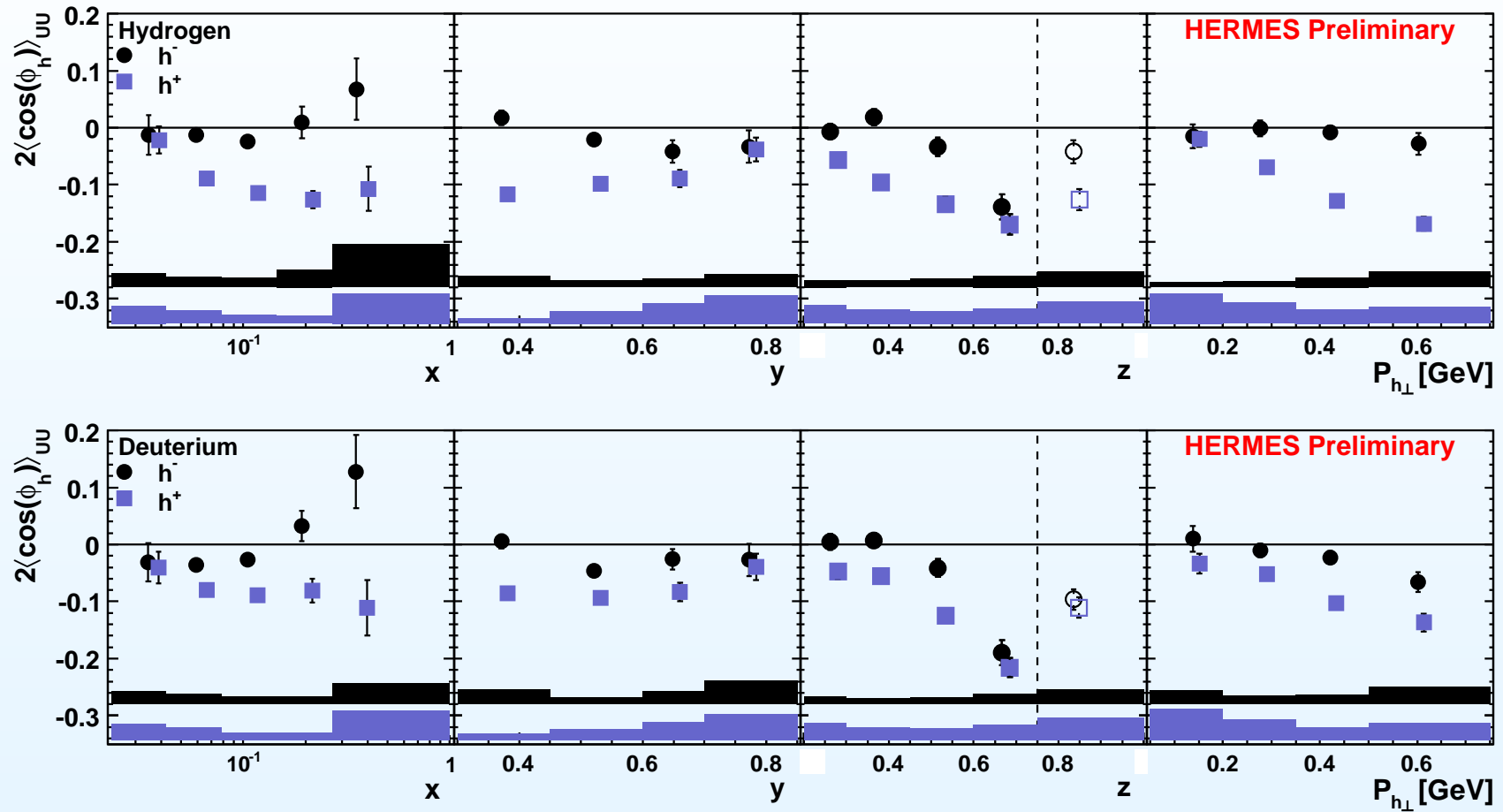


model by Barone, Prokudin, Ma, [Phys.Rev.D78:045022,2008](#)



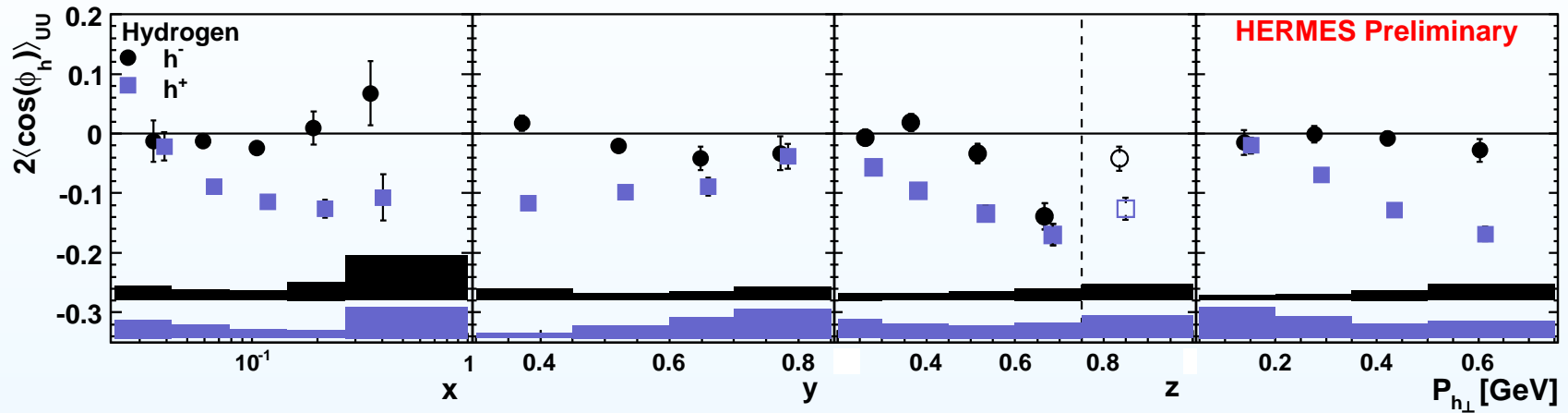
including Boer-Mulders and Cahn effect (twist-four contribution)

Results for $2 \langle \cos(\phi) \rangle_{UU}$:

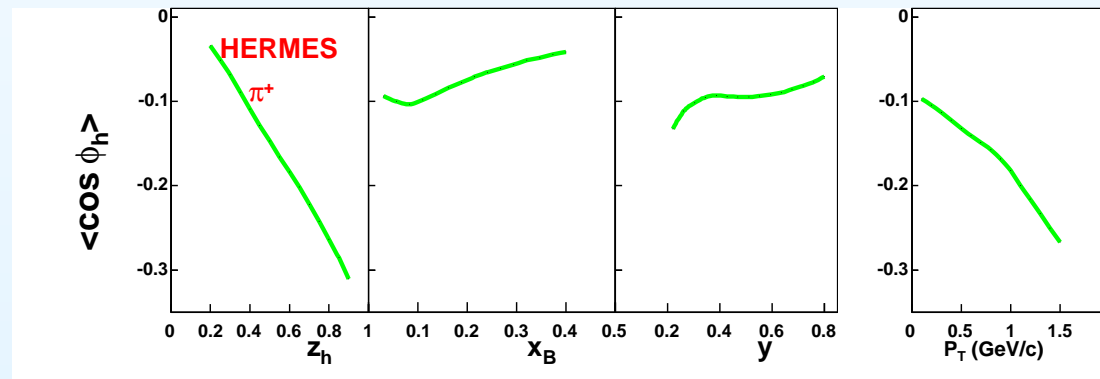


- almost zero for h^-
- significantly negative for h^+

Results for $2 \langle \cos(\phi) \rangle_{UU}$:



prediction by Anselmino et al., [Eur.Phys.J.A31:373-381,2007](#):

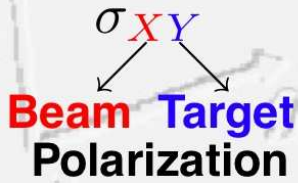


- quark-flavour dependent $\langle p_T \rangle$?
- significant Boer-Mulders contribution?

Towards the full cross-section measurement:

One-hadron production

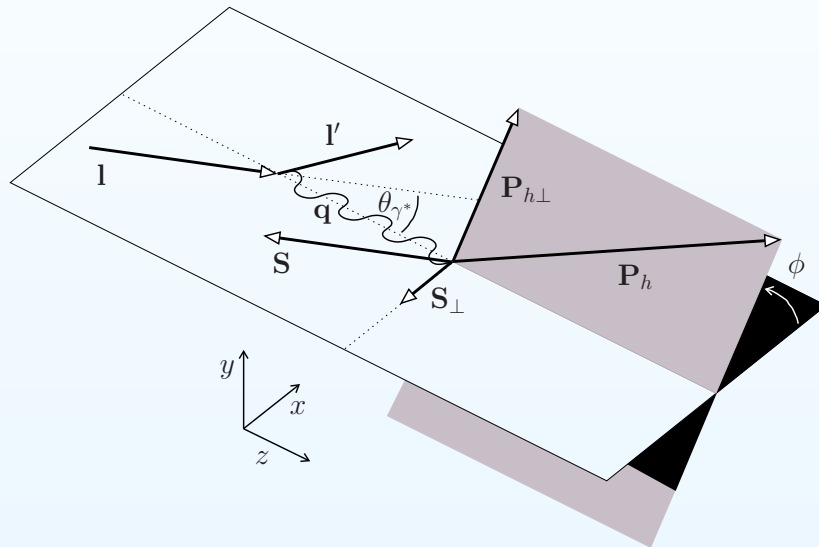
$$\begin{aligned}
 d\sigma = & d\sigma_{UU}^0 + \cos 2\phi d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi d\sigma_{LU}^3 \\
 & + S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left[d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right] \right\} \\
 & + S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \frac{1}{Q} \right. \\
 & \quad \left. + \frac{1}{Q} (\sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12}) \right. \\
 & \quad \left. + \lambda_e \left[\cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} (\cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15}) \right] \right\}
 \end{aligned}$$



Longitudinal single-spin asymmetries:

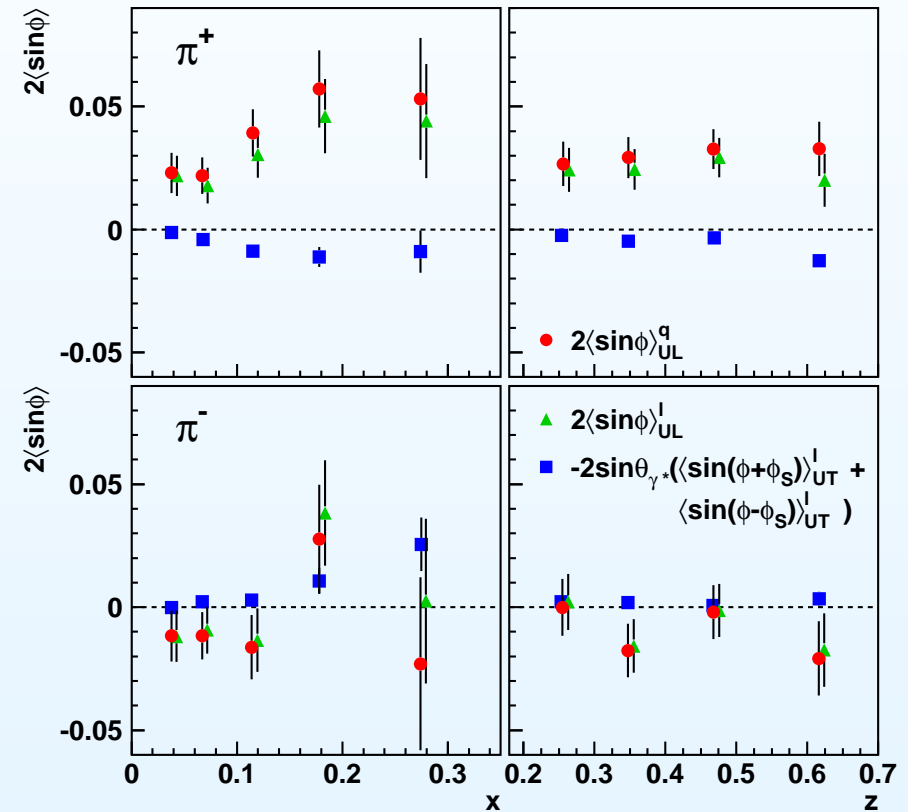
mixing of azimuthal moments:

evidence for subleading twist SSA:



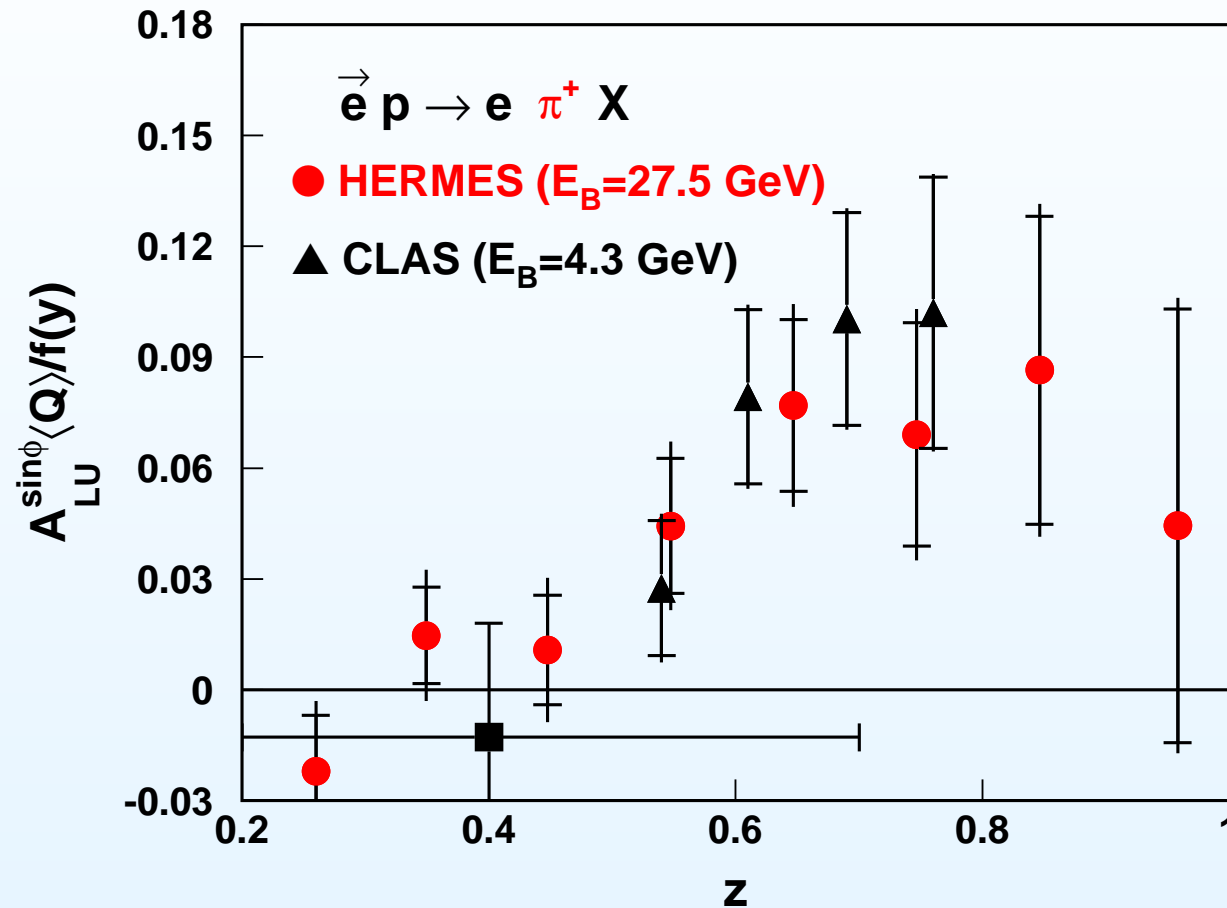
target spin axis w.r.t.:

- virtual photon axis ($q_{L,T}$)
- lepton beam axis ($l_{\parallel,\perp}$)



$$\langle \sin \phi \rangle_{UL}^q = \langle \sin \phi \rangle_{UL}^l + \sin \theta_{\gamma^*} \left(\langle \sin \phi + \phi_S \rangle_{UT}^l + \langle \sin \phi - \phi_S \rangle_{UT}^l \right)$$

Longitudinal beam-spin asymmetry:

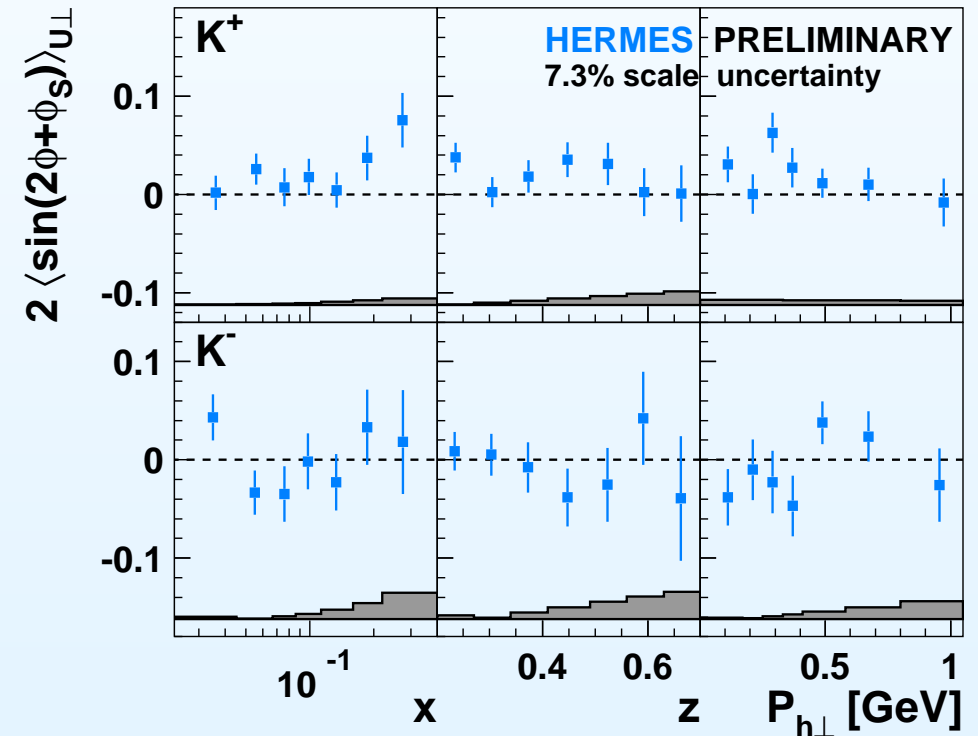
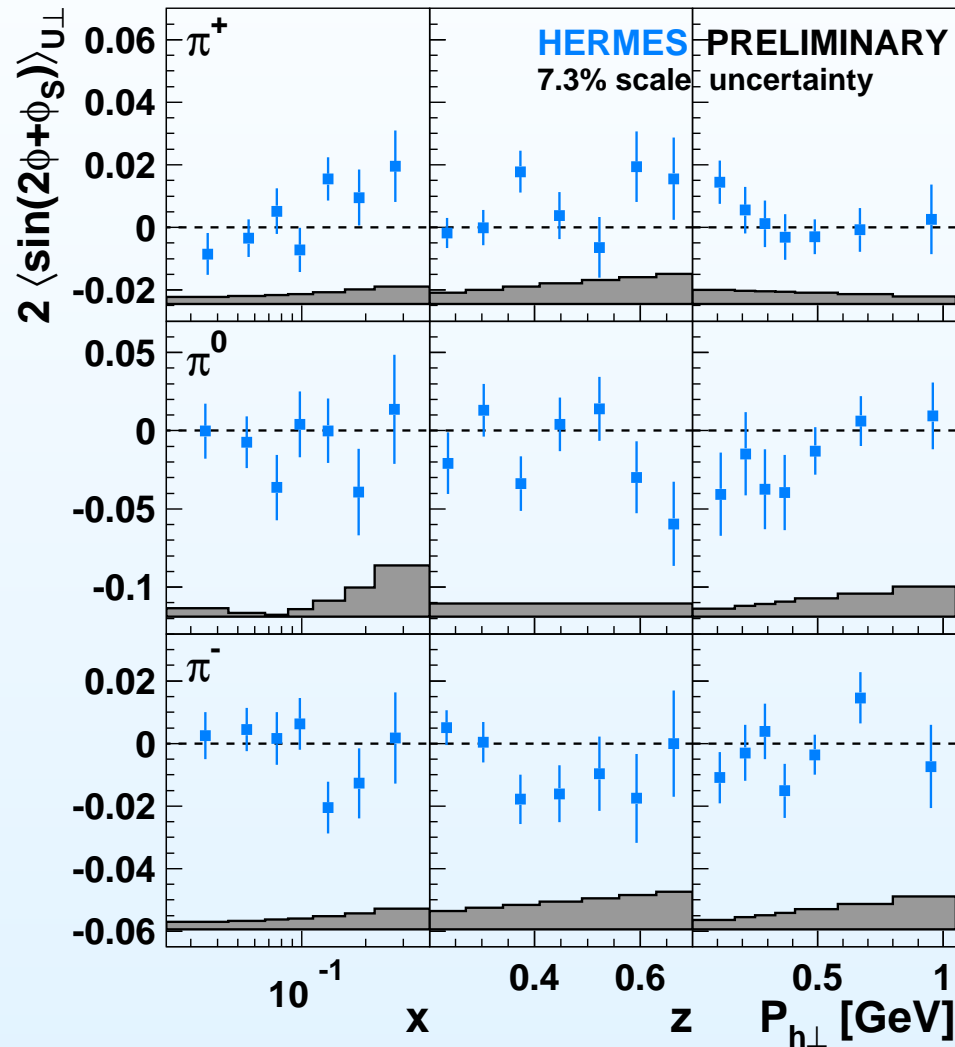


- good agreement with CLAS
- sensitive to $E(x)$ (but difficult to separate)

The $\langle \sin(2\phi + \phi_S) \rangle_{U\perp}$ Fourier component:

expected to scale as:

$$\frac{1}{2} \sin \theta_{\gamma^*} \langle \sin(2\phi) \rangle_{UL} \approx 0.01$$



The $\langle \sin(\phi_S) \rangle_{U\perp}$ Fourier component:

$$F_{UT}^{\sin \phi_S} = \frac{2M}{Q} c \left\{ \begin{aligned} & \left(x f_T D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z} \right) \\ & - \frac{\mathbf{k}_T \mathbf{p}_T}{2MM_h} \left[\left(x h_T H_1^\perp + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} \right) \right. \\ & \left. - \left(x h_T^\perp H_1^\perp - \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z} \right) \right] \end{aligned} \right\}$$

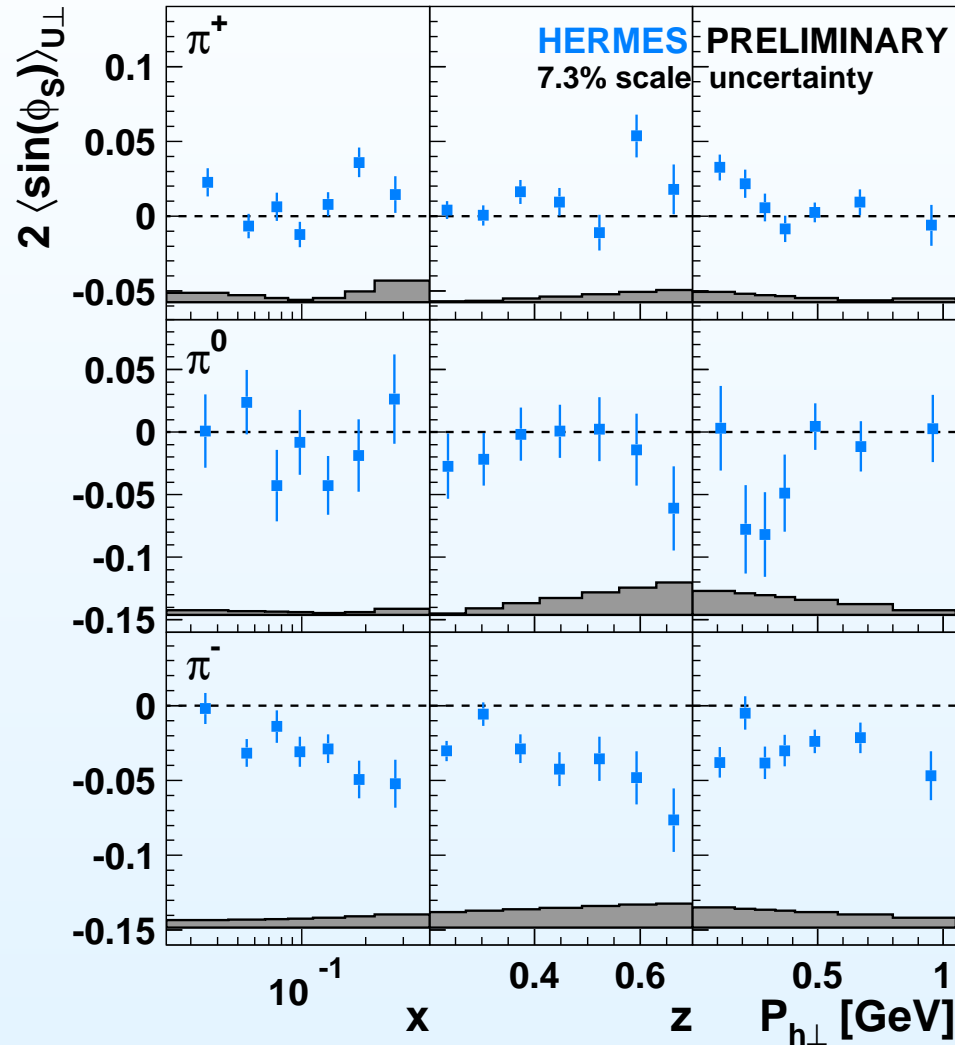
- using relations between T-even functions:

$$x h_T = x \tilde{h}_T - h_1 + \frac{\mathbf{p}_T^2}{2M^2} h_{1T}^\perp + \frac{m}{M} g_{1T}$$

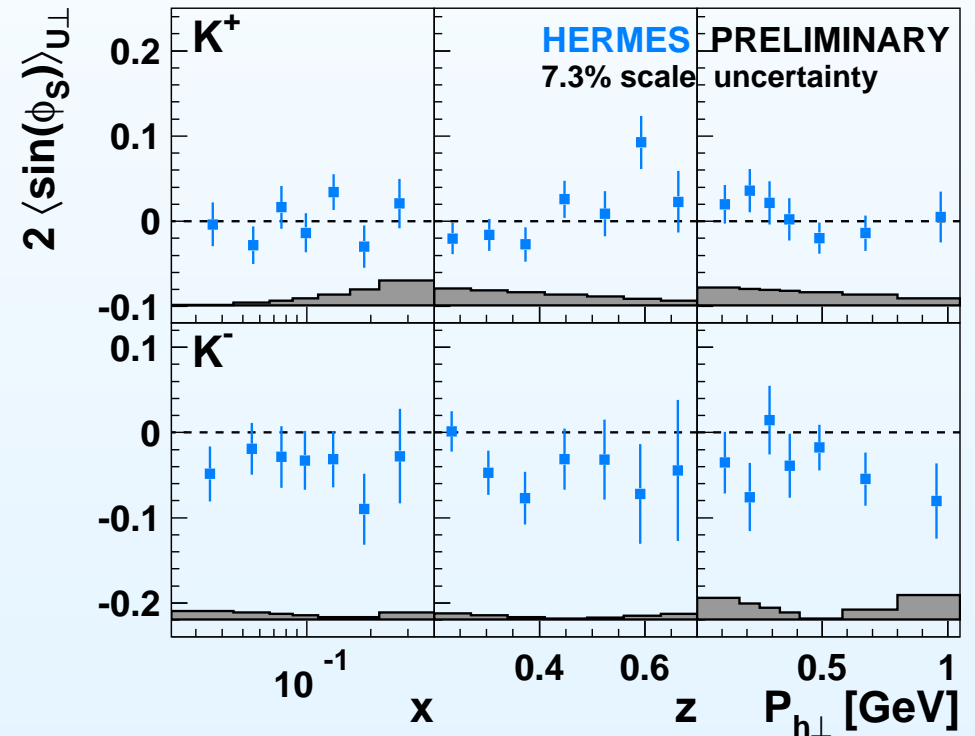
$$x h_T^\perp = x \tilde{h}_T^\perp + h_1 + \frac{\mathbf{p}_T^2}{2M^2} h_{1T}^\perp$$

- and the Wandzura-Wilczek approximation $\Rightarrow F_{UT}^{\sin \phi_S} \propto F_{UT}^{\sin(\phi + \phi_S)}$

The $\langle \sin(\phi_S) \rangle_{U\perp}$ Fourier component:



shape similar to
Collins amplitudes
expected

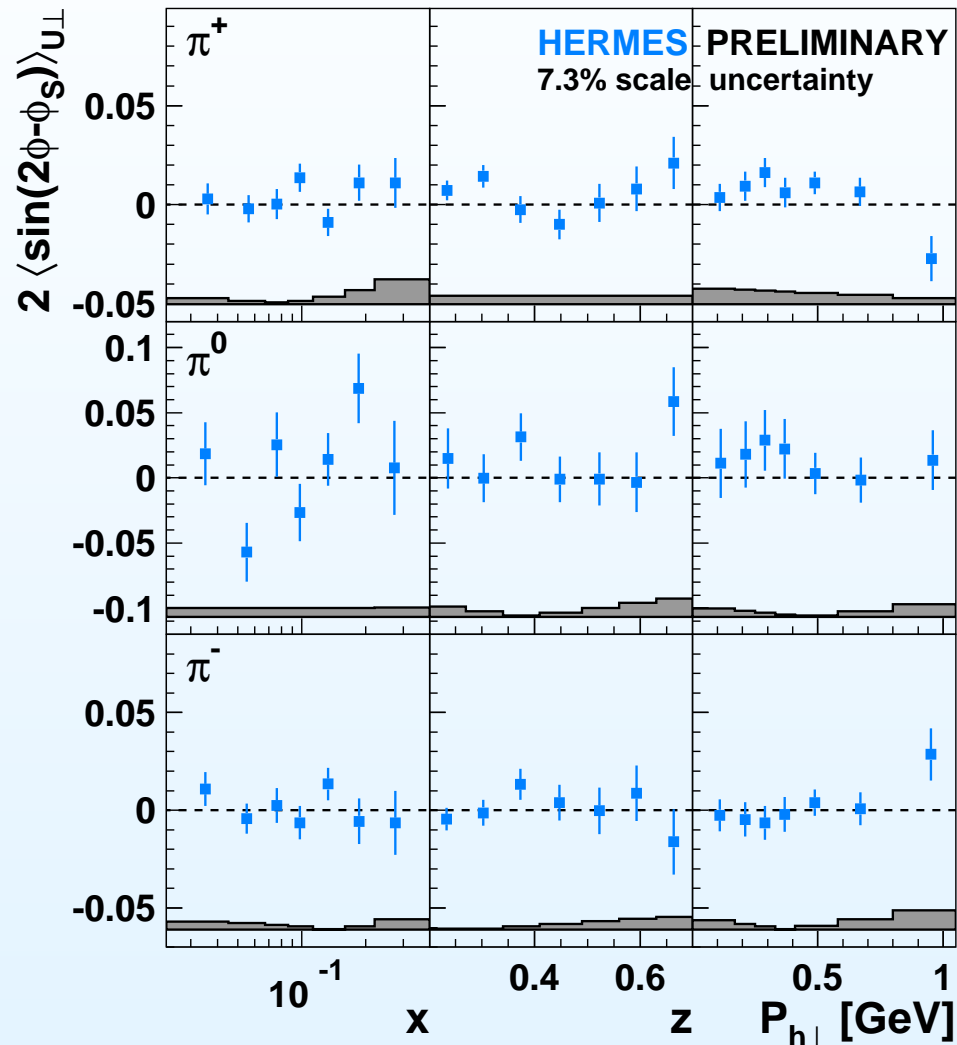


The $\langle \sin(2\phi - \phi_S) \rangle_{U\perp}$ Fourier component:

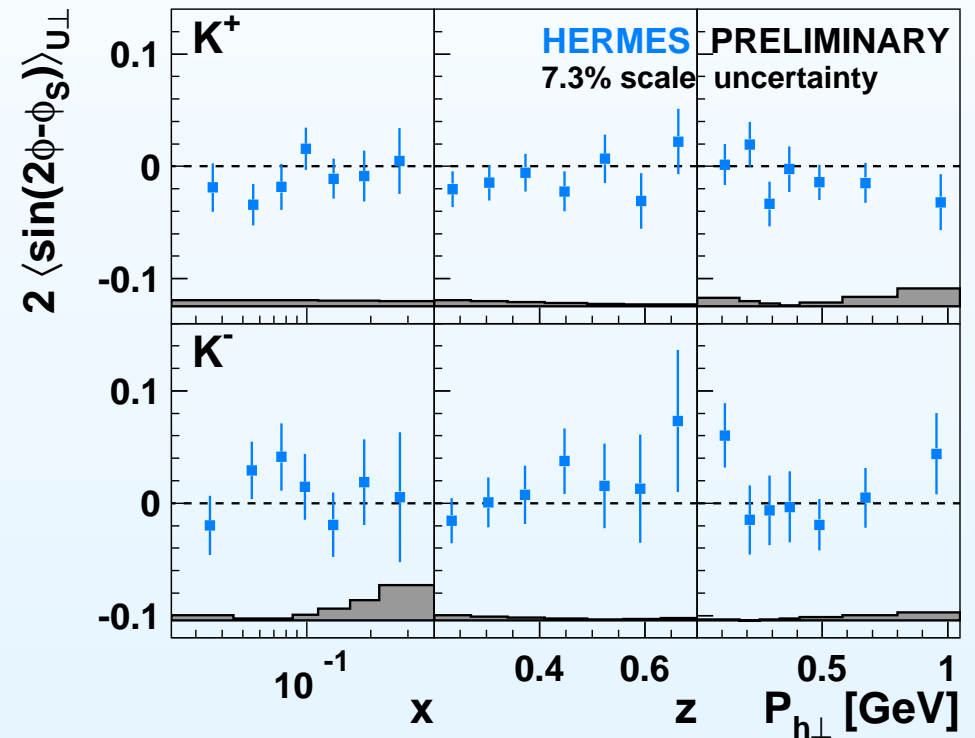
$$\begin{aligned}
 F_{UT}^{\sin(2\phi_h - \phi_S)} &= \frac{2M}{Q} \mathcal{C} \left\{ \frac{2(\hat{h}\mathbf{p}_T)^2 - \mathbf{p}_T^2}{2M^2} \left(x f_T^\perp D_1 - \frac{M_h}{M} h_{1T}^\perp \frac{\tilde{H}}{z} \right) \right. \\
 &\quad - \frac{2(\hat{h}\mathbf{k}_T)(\hat{h}\mathbf{p}_T) - \mathbf{k}_T \mathbf{p}_T}{2MM_h} \left[\left(x h_T H_1^\perp + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} \right) \right. \\
 &\quad \left. \left. + \left(x h_T^\perp H_1^\perp - \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z} \right) \right] \right\}
 \end{aligned}$$

- $F_{UT}^{\sin(\phi \pm \phi_S)}$ expected to scale as $P_{h\perp}$
- $F_{UT}^{\sin(2\phi - \phi_S)}$ expected to scale as $(P_{h\perp})^2$
 ↪ **suppressed w.r.t. Collins and Sivers amplitudes**

The $\langle \sin(2\phi - \phi_S) \rangle_{U\perp}$ Fourier component:



suppressed w.r.t.
Collins and Sivers amplitudes

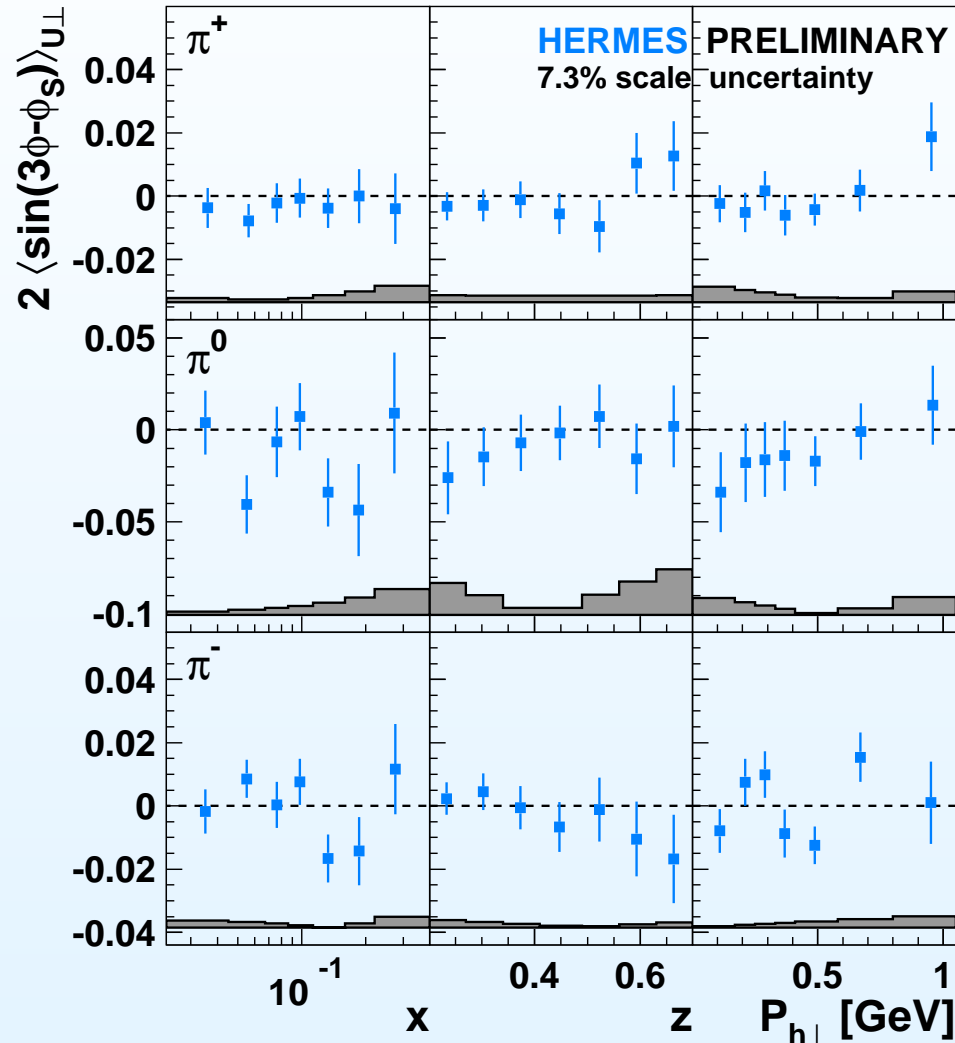


The $\langle \sin(3\phi - \phi_S) \rangle_{U\perp}$ Fourier component:

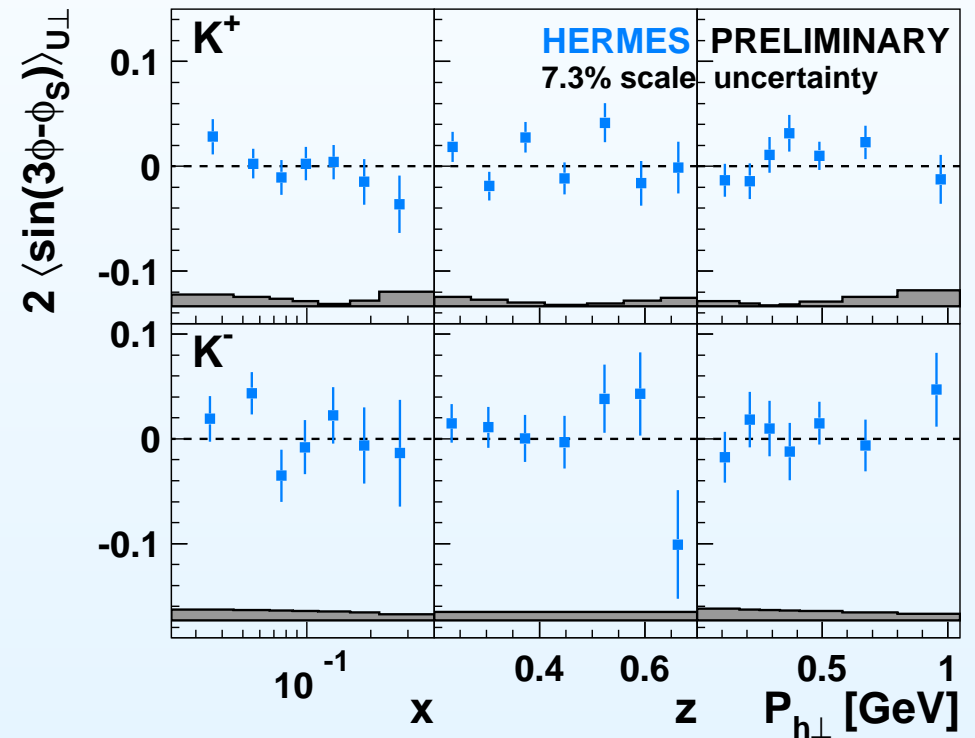
$$F_{UT}^{\sin(3\phi_h - \phi_S)} = C \left[\frac{2(\hat{h} \cdot \mathbf{p}_T)(\mathbf{p}_T \cdot \mathbf{k}_T) + \mathbf{p}_T^2(\hat{h} \cdot \mathbf{k}_T) - 4(\hat{h} \cdot \mathbf{p}_T)^2(\hat{h} \cdot \mathbf{k}_T)}{2M^2 M_h} h_{1T}^\perp H_1^\perp \right]$$

- leading-twist $F_{UT}^{\sin(3\phi - \phi_S)}$ sensitive to pretzelosity h_{1T}^\perp
- $F_{UT}^{\sin(\phi \pm \phi_S)}$ expected to scale as $P_{h\perp}$
- $F_{UT}^{\sin(2\phi - \phi_S)}$ expected to scale as $(P_{h\perp})^2$
- $F_{UT}^{\sin(3\phi - \phi_S)}$ expected to scale as $(P_{h\perp})^3$
↳ **suppressed w.r.t. Collins and Sivers amplitudes**

The $\langle \sin(3\phi - \phi_S) \rangle_{U\perp}$ Fourier component:



suppressed w.r.t.
Collins and Sivers amplitudes



In a nutshell:

- investigation of $\sigma_{UU}, \sigma_{UL}, \sigma_{UT}, \sigma_{LU}$
- significant $2 \langle \cos(\phi) \rangle_{UU}$ and $2 \langle \cos(2\phi) \rangle_{UU}$ amplitudes for hydrogen and deuterium target
 - ↳ sensitivity to Boer-Mulders function
- (most) precise data on a transversely polarised hydrogen target
- significant Collins amplitudes for π -mesons
 - ↳ enables quantitative extraction of transversity distribution
- significant Sivers amplitudes for π^+, π^0, K^+ and K^-
 - ↳ clear (and first) evidence of a naive-T-odd parton distribution
 - ↳ enables quantitative extraction of the Sivers function
- first evidence for a naive-T-odd dihadron fragmentation function
 - ↳ provides alternative probe for transversity distribution